# Working Paper Series 

# Manipulation of Cursed Beliefs in Online Reviews 

Ludmila Matysková<br>Jan Šípek

ISBN 978-80-7343-393-2 (Univerzita Karlova, Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-422-8 (Národohospodářský ústav AV ČR, v. v. i.)

# Manipulation of Cursed Beliefs in Online Reviews* 

Ludmila Matysková ${ }^{\dagger}$

Jan Šípek ${ }^{\dagger}$

April, 2017


#### Abstract

Consumer reviews may have perverse effects, including delays of adoption in new products of unknown quality when consumers are boundedly rational. When consumers fail to take into account that past reviewers self-select to purchases, a monopolist may manipulate the posterior beliefs of consumers who observe the reviews, because the product price determines the self-selection bias. The monopolist will charge a relatively high price because the positive selection of the early adopters increases the quality reported in the reviews.


Key Words: Cursed Equilibrium, Online Social Learning, Two-Sided Learning JEL classification: D42, D82, D83, L15

[^0]
## 1 Introduction

This paper challenges the general desirability of online reviews for experience products for the society. When consumers have heterogeneous preferences, each review posted reflects both the product's quality and the reviewer's idiosyncratic taste. With self-selection to buying decisions, reviews become a biased signal on unknown quality. If consumers do not correct for the self-selection, a monopolist may manipulate the posterior beliefs about the quality, because the product price determines self-selection bias. The monopolist faces a trade-off between higher demand today and higher posterior beliefs tomorrow: a higher price today lowers current demand, but intensifies self-selection to buying (only people with high enough idiosyncratic expectation/taste will buy), resulting in higher posterior beliefs tomorrow. We show that the monopolist charges a strictly higher initial price compared to myopic initial price in a setting without reviews, in order to increase bias in the posterior beliefs.

We present a 2-period model, where a monopolist dynamically prices a new experience product of a quality initially unknown to the monopolist and consumers. Updating of beliefs occurs by observing reviews that are truthfully and automatically posted by all past buyers. While the monopolist is rational, consumers have heterogeneous preferences and are boundedly rational - they do not take into account that reviewers self-select to purchasing decisions-as in the cursed equilibrium of Eyster and Rabin (2005). The bounded rationality is motivated by Li and Hitt (2008)'s empirical analysis of online book reviews: this suggests that consumers update beliefs as if the reviews reflected their own preferences, despite preference heterogeneity. In cursed equilibrium, agents correctly predict the distribution of actions, but neglect how these actions are correlated with other players' private information. Laboratory experiments (Parloura et al., 2007) as well as real world examples (Brown et al., 2012) show behavior consistent with cursed equilibrium. We offer another application of such boundedly-rational behavior.

This paper is related to literature on the strategic manipulation of the social learning (SL) process. Liu and Schiraldi (2012), Bhalla (2013), and Bose et al. (2006,
2008) analyze a setting where consumers have homogeneous preferences and private information about the unknown value of the product. Pricing decisions may then screen private information. In contrast, our consumers hold identical ex-ante information about quality, but have heterogeneous preferences. The learning does not come from observing purchasing decisions (revealing ex-ante private information) but from observing reviews (revealing ex-post satisfaction). The pricing decision then affects bias in the learning.

In a specific setting of online reviews, Crapis et al. (2015), and Ifrach et al. (2015) characterize equilibrium dynamics with rational and boundedly rational consumers, respectively. Rather than asking what the dynamics of the learning process are, we ask what distortions in pricing decisions are induced by the existence of reviews. Ifrach et al. (2015) show that 'the optimal dynamic pricing strategy charges a lower price than the corresponding myopic policy, which ignores the effect of pricing on the SL process', since a low price speeds up learning. In contrast, we suggest that the review system also generates incentives for charging a higher initial price than the corresponding initial myopic price.

In a similar setting to ours, Papanastasiou et al. (2014) show that a monopolist, constrained to charging a fixed price, may deliberately under-supply the early demand in order to increase bias in the consumers' posterior beliefs. However, no optimality of such under-supplies remains under dynamic pricing. We analyze a similarly unconstrained model.

In the framework of online reviews, firms may post fake reviews to bias consumers' beliefs (Dellarocas, 2006; Mayzlin et al., 2014). We show that even if we abstract from such practices, the monopolist is still able to manipulate consumers' beliefs using another tool: the price.

## 2 Model

A monopolist is selling an experience product of unknown quality, with zero marginal costs, over two periods. The monopolist maximizes total undiscounted expected profit by setting price $p_{t} \geq 0$ in period $t \in\{1,2\}$.

Each period $t$, a continuum of 1-period-lived consumers of mass one enters the market (unsatisfied demand is not carried over to the next period). Each consumer $i$ purchases the product whenever her expected utility $\mathbb{E}\left[u\left(a, \theta_{i}, p_{t}\right) \mid \theta_{i}, p_{t}\right]$ from purchase is nonnegative, where

$$
\begin{equation*}
u\left(q, \theta_{i}, p_{t}\right)=q+\theta_{i}-p_{t}, \tag{1}
\end{equation*}
$$

$q$ is an unknown common quality, and $\theta_{i}$ is a privately known idiosyncratic taste. Upon purchase, the consumer learns $q$ and automatically posts her experienced quality $q+\theta_{i}$ in a public review.

Information: $q$ is initially unknown to the monopolist and consumers. Common prior belief is $q \sim N\left(\mu, \sigma^{2}\right)$. Privately known taste parameters $\theta_{i}$ are iid random variables from uniform distribution on $[-\varepsilon, \varepsilon], \varepsilon>0$. We denote the random variables and their realizations by the same symbol. In period 2 , all agents observe the average of past-period reviews $\bar{r}=\mathbb{E}_{\theta}\left[q+\theta \mid \mu+\theta-p_{1} \geq 0\right]$, if there are any, as well as $p_{1}$ before taking action.

Consumers have the cursed beliefs of Eyster and Rabin (2005): each consumer correctly predicts the distribution of other consumers' actions, but do not take into account how these actions are correlated with idiosyncratic tastes. They incorrectly believe that others purchase the product randomly (with actual unconditional probability of purchase) irrespective of their taste, rather than in a way specified by their taste-contingent strategy ${ }^{1}$. The monopolist is rational and aware of the bounded ra-

[^1]tionality of consumers. Note that since all quality-relevant information is public, there is no signaling issue.

Timing: Nature chooses $q$. Consumers enter the market and privately learn their $\theta_{i}$ 's. The monopolist sets $p_{1}$. Consumers make their purchasing decisions. Anyone buying learns $q$ and posts a review $q+\theta_{i}$. Period 1 ends and all consumers leave the market. New consumers enter the market and privately learn their $\theta_{i}$ 's. The average review $\bar{r}$ from the previous period and $p_{1}$ become public. The monopolist sets $p_{2}$. Consumers make purchasing decisions. The game ends.

### 2.1 Cursed posterior beliefs

## Self-selection in period 1

If $p_{1} \leq \mu+\varepsilon$, consumer $i$ from period 1 buys iff $\theta_{i} \geq \hat{\theta}_{1}\left(p_{1}\right)$, where the threshold consumer $\widehat{\theta}_{1}\left(p_{1}\right)$ is

$$
\widehat{\theta}_{1}\left(p_{1}\right)= \begin{cases}-\varepsilon & p_{1} \leq \mu-\varepsilon  \tag{2}\\ p_{1}-\mu & \mu-\varepsilon<p_{1} \leq \mu+\varepsilon\end{cases}
$$

If $p_{1}>\mu+\varepsilon$, nobody purchases the product.
Given the realization of quality $q$, the average review $\bar{r}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)$, if there is any, is

$$
\begin{equation*}
\bar{r}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)=q+\mathbb{E}\left[\theta \mid \theta \geq \widehat{\theta}_{1}\left(p_{1}\right)\right] . \tag{3}
\end{equation*}
$$

## Rational (Bayesian) updating

Rational agents would account for self-selection. If past reviews exist, they 'de-bias' them appropriately, and recover the realized product quality by taking the inverse of (3). Otherwise, the posterior beliefs coincide with priors (since there is no new information).

## Cursed updating

Consumers mistakenly believe that reviews reflect the opinions of an unbiased random sample of the population, i.e., that $\bar{r}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)=\mathbb{E}[q+\theta]=q$, even though it is determined by (3). If past reviews exist, the cursed posterior belief is

$$
\begin{equation*}
q_{c}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)=\bar{r}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right) \stackrel{(3)}{=} q+\underbrace{\mathbb{E}\left[\theta \mid \theta \geq \widehat{\theta}_{1}\left(p_{1}\right)\right]}_{\text {bias }(\geq 0)} . \tag{4}
\end{equation*}
$$

Otherwise, the posterior beliefs coincide with priors (since there is no new information).

Since the monopolist is rational, her posterior beliefs are unbiased. The consumers' cursed posterior beliefs are unbiased only if the demand in period 1 was unity or zero. The monopolist endogenously manipulates the size of the bias through price $p_{1}$ (determining the intensity of the self-selection).

## 3 Inflation of price

## Definition 1. Equilibrium

An equilibrium is the monopolist's pair of prices $\left\{p_{1}^{*}, p_{2}^{*}\left(q, p_{1}^{*}\right)\right\}$ and consumers' pair of threshold tastes $\left\{\widehat{\theta}_{1}^{*}\left(p_{1}^{*}\right), \widehat{\theta}_{2}^{*}\left(q, p_{1}^{*}\right)\right\}$ such that:

1. In period 2, given $p_{1}^{*}, q$, and
(a) given $p_{2}^{*}\left(q, p_{1}^{*}\right)$, a consumer $i$ purchases the product iff $\theta_{i} \geq \widehat{\theta}_{2}^{*}\left(q, p_{1}^{*}\right)$;
(b) given $\widehat{\theta}_{2}^{*}\left(q, p_{1}^{*}\right), p_{2}^{*}\left(q, p_{1}^{*}\right)$ maximizes the second-period monopolist's profit.
(c) Consumers' posterior belief is given by (4) for $p_{1}=p_{1}^{*}$.
2. In period 1,
(a) given $p_{1}^{*}$, a consumer $i$ purchases the product iff $\theta_{i} \geq \widehat{\theta}_{1}^{*}\left(p_{1}^{*}\right)$;
(b) given a second-period subgame equilibrium for any $q$ (point 1.) and given $\widehat{\theta}_{1}^{*}\left(p_{1}\right), p_{1}^{*}$ maximizes the total undiscounted expected profit.

If $\mu \leq-\varepsilon$, nobody buys at any positive price. If $3 \varepsilon \leq \mu$, heterogeneity is relatively small, compared to $\mu$. The monopolist may find it optimal to satisfy the whole demand of period 1 , yielding no bias in $q_{c}$. A1 ensures a nontrivial case, where the monopolist always sells to a strict subset of consumers in period 1 (heterogeneity matters relatively).

Assumption 1. (A1): $-\varepsilon<\mu<3 \varepsilon$.

Comparing $p_{1}^{*}$ to its counterpart charged in a setting without review system, we establish the main result.

Proposition 1. Let A1 hold. Then the optimal first-period price is strictly higher in the setting with the reviews than without them.

Proof. Appendix.

## 4 Conclusion

This paper illustrates that the presence of consumer reviews may generate undesirable incentives. A monopolist selling an experience good can manipulate anticipations of the product's quality, even with truthful reviews. Consumers with the heterogeneous preferences and cursed beliefs of Eyster and Rabin (2005) fail to take into account that past consumers self-select themselves into purchasing decisions, leading to excessively high anticipation of quality. Since the higher the price is, the higher is the self-selection bias, the monopolist can exacerbate this error by increasing the price. The monopolist charges a higher initial price than the corresponding myopic price which ignores the effect of pricing on learning. The presence of reviews may thus slow down the diffusion of new products. Addressing the motivation to post reviews, or the reliability of reviews based on their sample size (currently absent due to the continuum of consumers assumption) are some possible extensions of the model.

## References

Bhalla, M. (2013), "Waterfall versus Sprinkler product launch strategy: Influencing the herd", The Journal of Industrial Economics 61(1), 138-165.

Bose, S., Orosel, G., Ottaviani, M., and Vesterlund, L. (2006), "Dynamic monopoly pricing and herding", RAND Journal of Economics 37(4), 910-928.

Bose, S., Orosel, G., Ottaviani, M., and Vesterlund, L. (2008), "Monopoly pricing in the binary herding model", Economic Theory 37, 203-241.

Brown, A.L., Camerer, C.F., and Lovallo, D. (2012), "To review or not to review? Limited strategic thinking at the movie box office", American Economic Journal: Microeconomics 4(2), 1-26.

Crapis, D., Ifrach, B., Maglaras, C., and Scarsini, M. (2015), "Monopoly pricing in the presence of social learning", NET Institute Working Paper No. 12-01.

Dellarocas, C. (2006), "Strategic manipulation of Internet opinion forums: Implications for consumers and firms", Management Science 52(10), 1577-1593.

Eyster, E., and Rabin, M. (2005), "Cursed equilibrium", Econometrica 73(5), 1623-1672.

Hanoch, G., and Levy, H. (1969), "The efficiency analysis of choices involving risk", Review of Economic Studies 36(3), 335-346.

Ifrach, B., Maglaras, C., and Scarsini, M. (2015), "Bayesian social learning from consumer reviews", available at SSRN: http://ssrn.com/abstract=2293158 .

Levy, H. (2015), Stochastic dominance: Investment decision making under uncertainty, Springer.

Li, X., and Hitt, L.M. (2008), "Self-selection and information role of online product reviews", Information System Research 19(4), 456-474.

Liu, T., and Schiraldi, P. (2012), "New product launch: herd seeking or herd preventing?", Economic Theory 51(3), 627-648.

Mayzlin, D., Dover, Y., and Chevalier, J. (2014), "Promotional reviews: An empirical investigation of online review manipulation", American Economic Review 104(8), 2421-2455.

Papanastasiou, Y., Bakshi, N., and Savva, N. (2014), "Scarcity strategies under quasi-Bayesian social learning", London Business School, working paper.

Parloura, C.A., Prasnikarb, V., and Rajan, U. (2007), "Compensating for the winner's curse: Experimental evidence", Games and Economic Behavior 60(2), 339-356.

## Appendix

## Proof of Proposition 1

Proof. From the ex-ante point of view, $q \sim N\left(\mu, \sigma^{2}\right)$. The monopolist's maximization problem is

$$
\begin{equation*}
\max _{p_{1} \in[\mu-\varepsilon, \mu+\varepsilon)} p_{1} \frac{\varepsilon-\left(p_{1}-\mu\right)}{2 \varepsilon}+\mathbb{E}\left[\Pi_{2}^{*}\left(q_{c}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)\right)\right] \tag{5}
\end{equation*}
$$

where

$$
\Pi_{2}^{*}\left(q_{c}\right)= \begin{cases}0 & q_{c}<-\varepsilon  \tag{6}\\ \left(\frac{1}{2 \varepsilon}\right)\left(\frac{q_{c}+\varepsilon}{2}\right)^{2} & -\varepsilon \leq q_{c} \leq 3 \varepsilon \\ q_{c}-\varepsilon & q_{c}>3 \varepsilon\end{cases}
$$

is the equilibrium second-period profit (found by backward induction), and the expectation is taken over $q_{c}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)$ given by (4). Given $p_{1} \in[\mu-\varepsilon, \mu+\varepsilon)$ and the distribution assumptions ${ }^{2}, q_{c}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right) \sim N\left(\mu_{u}\left(p_{1}\right), \sigma^{2}\right)$ ex-ante, where

$$
\begin{equation*}
\mu_{u}\left(p_{1}\right)=\mu+\underbrace{\frac{1}{2}\left(p_{1}-\mu+\varepsilon\right)}_{\text {bias }} \tag{7}
\end{equation*}
$$

[^2]The slope of (5) is

$$
\begin{equation*}
\frac{\mu-2 p_{1}+\varepsilon}{2 \varepsilon}+\frac{\mathrm{d}}{\mathrm{~d} p_{1}} \mathbb{E}_{q_{c}}\left[\Pi_{2}^{*}\left(q_{c}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)\right)\right] . \tag{8}
\end{equation*}
$$

We show that the slope (8) is strictly positive for $p_{1} \in\left[\mu-\varepsilon, \frac{\mu+\varepsilon}{2}\right]$, which implies that the solution to (5) is $p_{1}^{*}>\frac{\mu+\varepsilon}{2}$.

A1 ensures that $\frac{\mu+\varepsilon}{2} \in[\mu-\varepsilon, \mu+\varepsilon)$. Furthermore,

1. $\frac{\mu-2 p_{1}+\varepsilon}{2 \varepsilon}>0$ for $p_{1}<\frac{\mu+\varepsilon}{2}$ and equals zero at $p_{1}=\frac{\mu+\varepsilon}{2}$.
2. $\frac{\mathrm{d}}{\mathrm{d} p_{1}} \mathbb{E}_{q_{c}}\left[\Pi_{2}^{*}\left(q_{c}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)\right)\right]>0$ for any $p_{1} \in(\mu-\varepsilon, \mu+\varepsilon)$ :
(a) Since $q_{c}$ is Gaussian with variance independent of $p_{1}$, and mean $\mu_{u}\left(p_{1}\right)$ increasing in $p_{1}$, the ex-ante distribution of posterior beliefs first-order stochastically increases in $p_{1}$ for $p_{1} \in[\mu-\varepsilon, \mu+\varepsilon)(\operatorname{Levy}(2015))$.
(b) (6) is a non-decreasing function of $q_{c}$, strictly increasing on some intervals. First-order stochastic dominance implies $\frac{\mathrm{d}}{\mathrm{d} p_{1}} \mathbb{E}_{q_{c}}\left[\Pi_{2}^{*}\left(q_{c}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)\right)\right] \geq 0$. Gaussian distribution ${ }^{3}$ and Lemma 1 of Hanoch and Levy (1969) ensures that the inequality is strict, $\frac{\mathrm{d}}{\mathrm{d} p_{1}} \mathbb{E}_{q_{c}}\left[\Pi_{2}^{*}\left(q_{c}\left(q, \widehat{\theta}_{1}\left(p_{1}\right)\right)\right)\right]>0$.

In a setting without a review system, the monopolist's maximization problem in period 1 is $\max _{p_{1} \in[\mu-\varepsilon, \mu+\varepsilon)} p_{1} \frac{\varepsilon-\left(p_{1}-\mu\right)}{2 \varepsilon}$, resulting in optimal price $p_{1, a}^{*}=\frac{\mu+\varepsilon}{2}<p_{1}^{*}$.

[^3]
#### Abstract

Abstrakt

Když jsou spotřebitelé omezeně racionální, spotřebitelské recenze mohou mít zvrácené účinky zahrnující zpoždění v ujímání se nových produktů neznámé kvality. Pokud spotřebitelé opomíjejí, že u recenzentů došlo k samoselekci ohledně zakoupení či nezakoupení produktu, monopolista může manipulovat přesvědčení spotřebitelů ohledně kvality produktu aktualizované po přečtení recenzí, protože cena produktu určuje zkreslení dané samoselekcí (tzv. self-selection bias). Monopolista nastaví relativně vysokou cenu, protože pozitivní samoselekce u počátečních kupců zvýší reportovanou kvalitu v jejich následných recenzích.


## Working Paper Series

ISSN 1211-3298
Registration No. (Ministry of Culture): E 19443
Individual researchers, as well as the on-line and printed versions of the CERGE-EI Working Papers (including their dissemination) were supported from institutional support RVO 67985998 from Economics Institute of the CAS, v. v. i.

Specific research support and/or other grants the researchers/publications benefited from are acknowledged at the beginning of the Paper.
(c) Ludmila Matysková and Jan Šípek, 2017

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical or photocopying, recording, or otherwise without the prior permission of the publisher.

Published by
Charles University, Center for Economic Research and Graduate Education (CERGE) and
Economics Institute of the CAS, v. v. i. (EI)
CERGE-EI, Politických vězñů 7, 11121 Prague 1, tel.: +420 224005 153, Czech Republic.
Printed by CERGE-EI, Prague
Subscription: CERGE-El homepage: http://www.cerge-ei.cz
Phone: + 420224005153
Email: office@cerge-ei.cz
Web: http://www.cerge-ei.cz
Editor: Jan Zápal
The paper is available online at http://www.cerge-ei.cz/publications/working_papers/.
ISBN 978-80-7343-393-2 (Univerzita Karlova, Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-422-8 (Národohospodářský ústav AV ČR, v. v. i.)


[^0]:    *We thank Filip Matějka, Avner Shaked, Jakub Steiner, Jan Zápal, and participants at YEM 2016 in Brno for valuable comments. Matysková was supported by the Charles University, project GA UK No. 178715.
    ${ }^{\dagger}$ CERGE-EI, a joint workplace of Charles University and the Economics Institute of the Czech Academy of Sciences, Politických vězňů 7, 11121 Prague, Czech Republic; email: lmatysko@cerge-ei.cz (Matysková)

[^1]:    ${ }^{1}$ Eyster and Rabin (2005) parametrize the 'cursedness' by $\chi \in[0,1]$. Agents believe others use a type-contingent strategy with probability $1-\chi$ and act independently of their type otherwise. For simplicity, we set $\chi=1$, but our results hold $\forall \chi>0$.

[^2]:    ${ }^{2}$ Optimal price $p_{1}^{*} \in[\mu-\varepsilon, \mu+\varepsilon)$, since charging $p_{1} \geq \mu+\varepsilon$ or $p_{1}<\mu-\varepsilon$ is strictly dominated by $p_{1}=\mu-\varepsilon$.

[^3]:    ${ }^{3}$ If Gaussian distribution $A$ first-order stochastically dominates $B$, then $\operatorname{CDF}_{A}(x)<$ $\mathrm{CDF}_{B}(x) \forall x \in \mathbb{R}$, where CDF denotes cummulative distribution function (Levy, 2015).

