WHY MIXED QUALITIES MAY NOT SURVIVE AT EQUILIBRIUM: THE CASE OF VERTICAL PRODUCT DIFFERENTIATION

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Why Mixed Qualities May Not Survive at Equilibrium: The Case of Vertical Product Differentiation*

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Abstract

In the classical literature on vertical differentiation, goods are assumed to be single products each offered by a different firm and consumed separately one from another. This paper departs from the standard setup and explores the price competition in a vertically differentiated market where a firm's product is consumed not separately but in fixed one-to-one ratio with another complementary type of good supplied by a different producer. An optimal solution for market setting with two entrants of a type is proposed, to show that there could be an equilibrium at which the so-called "mixed-quality combinations", consisting of one high-quality good and one low-quality good each, remain unsold. For such an equilibrium to exist, it is sufficient the mixed-quality combinations to be at least as differentiated from the best as from the worst combination which retains its positive market share. Thus, the mixed-quality exclusionary outcome appears as a further form in which the well-known maximum-differentiation principle could be implemented in a multi-market setting. It provides a new explanation of the self-selection bias in consumption observed in some industries for complementary goods.

Keywords: complementary goods, vertical product differentiation, market foreclosure
JEL classification: L11, L13, L15

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1 Introduction

The principles that drive firms' behavior in a single vertically differentiated market are well-established in the existing industrial organization literature (Gabszewicz and Thisse (1979), Shaked and Sutton (1982) and Tirole (1988)). Little is known, however, about the way these principles hold in a broader multi-market environment. In the present paper, we make first steps in this direction by modeling two adjacent vertically differentiated markets for final goods consumed as complements.

Adjacent vertically differentiation markets are not uncommon. There are many examples of vertically differentiated goods that are complements to other goods which are also vertically differentiated. In this particular case, consumers always purchase them both in fixed one-to-one ratio even though they are sold by different firms, in different markets and at different prices. For instance, when buying a new flat or a house from a real-estate company, consumers also buy together with it furniture from the home-furnishing shop or order it from a cabinet-maker. Also, when going on a business trip, employees should be provided not only with transportation from an airline company but also with accommodation from a hotel at the destination of the trip.[1] Or, when buying a computer, user actually pays for a central-processing unit (CPU) manufactured by a computer hardware firm, screen from a TV producer, an operation platform from a system software provider, and application software from a number of application software developers.

One of the most interesting features of single vertically differentiated markets is the so-called maximum-differentiation principle that drives firms' optimal quality choice. Mussa and Rosen (1978) first show that an uninformed multi-product monopolist could benefit from broadening the range of the quality spectrum of its product line which would allow it to imperfectly price discriminate between consumers who have different tastes for quality and therefore heterogeneous willingness to pay for it. Since the consumers with a low willingness to pay are inclined to compromise with quality, they could be easily distinguished from the consumers with a high willingness to pay by strategic deterioration of the lower edge in the quality spectrum offered by the monopolist. Gabszewicz and Thisse (1979) apply the same principle to a duopoly market with single-product firms where consumers differ by income. They show that duopolists could gain from differentiating the quality of their products which leads to segmentation of the market and raises the market power of each duopolist in its respective segment. Maskin and Riley (1984) extend the model of Mussa and Rosen (1978) for the case of multiple-product monopolist facing non-unit consumption. Donnenfeld and White (1988) introduce a discrete heterogeneity of consumers to show that a multi-product monopolist would prefer to broaden the quality spectrum

[1] Note that transportation and accommodation are also demanded when going on a holiday trip but the two are mostly offered by the holiday agencies in a pre-selected bundle. In our paper, however, we are interested in studying only vertically differentiated markets where the consumer choice is in no way pre-selected by firms but comes out as an optimal consumers' decision given firms' optimal quality and price choices. For a thorough discussion of the competition and welfare effects of the existing practices of firms to restrict free consumer choice by providing them with pre-selected bundles of goods in the form of take-it-or-leave it offer, see Whinston (1990), Carlton and Waldman (2002), Nalebuff (2003a, 2003b).

of its product line upwards when the relationship between absolute and marginal willingness to pay for quality is allowed to be negative. Champshur and Rochet (1989) consider the case of a multi-product duopoly and prove that even though duopolists still prefer to broaden their quality spectrums there will always be a gap between their product lines at equilibrium. The competition relaxing effect of vertical product differentiation is further explored by Moorthy (1988), Choi and Shin (1992) and Wauthy (1996).

In this paper, we propose a model of two adjacent vertically differentiated markets to explore how the maximum-differentiation principle might drive outcomes in this new setting. We assume complementary goods to be consumed in fixed one-to-one proportion and supplied by single-product firms while consumers are not restricted in their choice how to combine the available goods from the two markets. Particularly, the solution of the model implies that a duopoly equilibrium exists at which firms choose not only to maximize the differentiation between their products within each market but also to charge prices at which the quality/price ratio of the less-differentiated combinations is not optimal for any of the consumers. That is, only the most differentiated combinations that consist of the two top-quality and the two bottom-quality goods will be sold, while the remaining combinations which we call 'mixed-quality combinations' will remain unsold.

In some vertically differentiated markets for complementary goods it is not uncommon to observe self-selection bias in consumers' choices so that only the combinations of goods with similarly ranked qualities and prices are purchased by consumers while the mixed-quality combinations are ignored. For instance, let us refer to the above mentioned examples. It would be very unusual to see consumers who buy an expensive luxurious house and furnish it with cheap do-it-yourself furniture. Or vice versa, normally consumers who buy small cheap flats do not equip them with high-design expensive furniture. Likewise, businessmen who order business-jet charter flights naturally choose to be accommodated in five-star hotels which is not the case of small-firm entrepreneurs or lower-level managers whose business-trip budget only allows them to travel in the business or even economy class of the regular airline flights. [2]

One straightforward explanation for the observed self-selection choice of consumers could be that the valuations for the both types of goods are identically distributed among consumers so that the buyers of the high-quality good in one of the markets at equilibrium also choose the high-quality good in the other market while the rest purchase the low-quality goods in both markets. Therefore, in the end, nobody finishes with a mixed-quality combination. However, this explanation is implied by the assumption that consumers choose each of the complementary goods in its single market (Kováč 2007).

Given that two goods are complements, it is perhaps too strong an assumption to make that consumers purchase them in single markets. This is our argument why in the present paper, we follow a different approach by assuming instead that consumers choose not from the goods available in each market but from the one-to-one combinations that could be formed out of the goods in both markets. In the setting with two duopoly

[2] We are grateful to Michael Kunin for suggesting both the house-furniture and the flight-hotel examples.

markets, there are four possible combinations of the two goods in each of the markets. The best combination is apparently given by the pair of the higher-quality goods while the worst combination involves the lower-quality goods only. The remaining two mixed-quality combinations are ranked second and third respectively, based on how substitutable are the qualities in one of the markets by the qualities in the other market while determining the qualities of the combinations in which they take part. To derive a determinable solution, we also introduce individual upper bound on the quality choice of each potential entrant as well as general lower bound on the quality choice of all firms in each market. Intuitively, the upper constraint on the quality choice of the firms could be considered as a contemporary maximal technological frontier which varies across firms whereas the lower restriction may represent the minimal social (safety or hygienic) requirements for a good to be acceptable for sale in a given market.

In the two-market setting that we suggest, each good participates in two of the possible combinations and therefore by choosing the price of its good a firm is also affecting the demand for the two possible combinations in which it takes part. If the combination of the two high-quality goods and the combination of the two low-quality goods are the only two combinations that have positive demand at equilibrium, this should not be only because maximum differentiation quality choices were made by the entrants in the both markets. In addition, it must be the case that the high-quality producers find it suboptimal to set prices low enough for their goods to be sold also in mixed-quality combinations with their low-quality counterparts from the other market.

This new approach that we apply allows us furthermore to identify how the characteristics of the well-known single-market duopoly equilibrium changes in the context of a two-market setting. The existing literature uses three key properties to characterize the equilibrium duopoly outcome in a single vertically differentiated market. We take these properties as a benchmark when analyzing the optimal solution of the two-market setting. Accordingly, before presenting our results, we will discuss below the single-market properties as initially described by Shaked and Sutton (1982, 1983).

First, Shaked and Sutton (1982) explore a simple setting where firms have zero unit production cost and consumer incomes follow a continuous uniform distribution, to show the positive correlation between the market segmentation and the dispersion in consumers' incomes. This property follows straight from the definition of vertical product differentiation which states that independent on how consumers differ by income they all rank unanimously the qualities of the available goods in the market. That is to say, no matter how low is the income of a consumer, she (or he) will always value and be willing to pay more for the top-ranked good than for the second-best ranked good. Similarly, her (or his) valuation will be higher for the second-best ranked good than for the third-best ranked good, and so on down the quality ranking of goods with which all consumers agree. The firm offering the top quality, therefore, can efficiently drive out from the market all the competitors and serve the whole market alone. For this purpose, it is sufficient to charge its good a price that exceeds the valuation of the lowest-income consumer for the second-best ranked good but is still below her (or his) reservation price for the top-quality good. Whether the top-quality firm will, however, find it optimal to charge such an exclusionary price would depend on how low is the lowest consumer income relative to the highest consumer income. The lower is the ratio between the two the more likely it would be for the market size to exceed the profit-maximizing scale of the top-quality firm. Then, there will be more space for further segmentation through accommodating the market entry of other lower-quality firms. In particular, Shaked and Sutton (1982) prove that if, and only if, the lowest consumer income is at most four times but not more than twice smaller than the largest consumer income, exactly two firms could have positive market shares at equilibrium.

Second, Shaked and Sutton (1982) demonstrate that the same condition that ensures at most two market entrants is also sufficient for the market to be covered, that is each consumer will buy the one or the other of the two available goods at the single duopoly equilibrium. Moreover, the result does not depend on the quality differentiation in the market. Quality difference positively affects only the prices of the two entrants but not their market shares.

Third, Shaked and Sutton (1983) derive a general condition for having the so-called finiteness property of the vertically differentiated markets, even in the case of positive unit production costs. Opposed to the classical (monopolistic competition) outcome of the representative-consumer model of market differentiation described by Chamberlin (1933), it states that when the entry barriers tend to zero in a vertically differentiated market, the number of firms with positive market share remains finite. For the finite upper bound on the number of entrants at equilibrium to exist, the difference in the price markups of any two firms whose goods have neighboring quality ranks should be larger than the difference in their unit production costs. Analogously to the case without production cost, the result again follows straight from the definitional assumption that there is unanimous consensus among consumers how the available goods should be ranked by quality. Accordingly, since the finite number of entrants is endogenously determined, Shaked and Sutton (1983) call the corresponding market structure 'natural oligopoly'.

In our paper, we start with analyzing the solution for the case when the number of entrants in one of the markets is exogenously restricted to one. For expositional convenience we identify the consumers by taste as in the classical model of Mussa and Rosen (1978). This does not affect the comparability of our results with the respective results of Shaked and Sutton (1982) because as it is shown by Tirole (1988) richer consumers are also more demanding so that consumers' identification by taste could be considered to be equivalent to that by income. Accordingly, we assume a dispersion of consumer tastes which satisfies the condition on the consumer income distribution, proven by Shaked and Sutton (1982) to be necessary and sufficient for having a covered-duopoly outcome in a single-market setting. Our results, however, suggest the existence of a unique non-covered monopoly equilibrium at the two-market setting instead. This outcome is also surprising in the context of the literature on market foreclosure (Whinston 1990). It implies that leveraging the market power from an adjacent market with a lower level of competition might appear not as a consequence but rather as a cause of the market exclusion of the competitors. The condition for having all but the best-quality entrant in the adjacent market blockaded is its quality choice upper bound to be at least nine-forth times larger than the upper bound on the quality choice of the entrant with the second-best good.

We further show that the difference in the market power of firms between the two adjacent markets is decisive for the ranking of the mixed-quality combinations given two entrants in each market. Due to the positive relationship between quality differentiation and the market power of firms, there is a clear dependence between the quality ranking of the goods in the more differentiated of the two markets and the ranking of the mixed-quality combinations in which these goods take part. That is, the mixed-quality combination based on the higher-quality good in the more differentiated market has higher rank than the mixed-quality combination based on the lower-quality good in the same market. This is demonstrated for three different possible relations between the qualities of any pair of available complementary goods with regard to how they define the quality of the combination they form. Particularly, we consider linear (perfectly substitutable qualities), multiplicative (non-perfectly substitutable qualities) and Leontief (perfectly complementary qualities) functional relationships between the combined goods' qualities.

Next, we analyze the solution for the case when the number of entrants in both markets is exogenously restricted to two. Our results imply that for having the mixed-quality combinations unsalable at equilibrium, it is sufficient for the quality of the second-best ranked combination to be at most equal to the median between the qualities of the best and the worst combinations. Then, the higher-quality firm in the more differentiated market does not benefit from decreasing the price of its good so low as is necessary for selling it as a part of the second-best ranked combination that is, together with the lower-quality good. We further show that the condition for having this equilibrium outcome could not hold when the qualities are linearly related because the higher-quality good perfectly substitutes its lower-quality counterpart in the second-best combination. The exclusionary outcome for the mixed-quality combinations, however, is proven to be feasible at equilibrium in the case of less substitutable (multiplicative and Leontief) relationships between the qualities of the combined complementary goods.

Additionally, we show that, distinct from the single-market solution, the dispersion of consumer tastes, for which Shaked and Sutton (1982) prove a covered duopoly outcome, is neither necessary nor sufficient for having both markets covered at equilibrium in the two-market setting. The reason is again the difference in quality differentiation between the two markets. The lower-quality firm in the more differentiated market has higher market power, which makes it less willing than its counterpart in the adjacent market to decrease its price in order to cover the market. Therefore, the consumers must be more demanding than at the single-market equilibrium to be still willing to pay the higher optimal price of the lowest-quality combination instead of giving it up for an inferior free non-market option. Furthermore, the larger the differentiation between the best and the worst combinations the higher the lowest consumer taste for quality should also be for the market to be covered at equilibrium. This implies that quality differentiation of the combinations has a relaxation effect on the competition of firms which resembles the well-recognized competition relaxing effect of the product differentiation in a single-market setting. Indeed, when firms can combine their goods with complementary goods from another vertically differentiated market, this extends their ability to gain market power through direct quality differentiation of their goods by also allowing them to implicitly restrict the salability of their mixed-quality combinations through exclusionary pricing.

Finally, we derive the conditions for effective deterrence of a potential third entrant in any of the two markets. It is also not independent on the quality choice of the firms. Namely, the upper bound on the quality choice of the entrant needs to be at least twice smaller than the upper bound of the higher-quality firm in the market targeted by the entrant. Otherwise, the quality of the best combination involving the good of the third entrant is so close to the quality of the worst combination of the incumbent goods that the suppliers of the latter find it optimal to accommodate the entrant at equilibrium by setting prices at which the lowest-income consumers buy the entrant's good.

The paper is organized as follows. Section 2 describes the model, section 3 presents the particular solution for the proposed subgame-perfect equilibrium and establishes gradually the sufficient conditions for it to hold. Section 4 summarizes the results and provides brief comment on their implications.

2 Description of the Model

Here we start with the introduction of a stylized model of a market for combinations consisting of two types of goods each offered independently by a separate firm. The two types of goods are denoted by A and B, respectively. Since goods are assumed to be be consensually ranked by quality, they can be identified by their rank starting from the best-quality good being ranked by 1, the second-best by 2 and so on, as shown below:

$$f\left(A_1\right) > f\left(A_2\right) > .... > f\left(A_m\right) > 0 \tag{1}$$

where $f\left(A_i\right)$ denotes the value that consumers assign to a mutually-agreed mix of characteristics of the product of type A based on which it is ranked at i-th place, $i = 1, ..., m$;

$$g\left(B_1\right) > g\left(B_2\right) > .... > g\left(B_n\right) > 0 \tag{2}$$

where $g\left(B_j\right)$ denotes the value that consumers assign to a mutually-agreed mix of characteristics of the product of type B based on which it is ranked at j-th place, $j = 1, ..., n$.

The assumption that the two types of goods are complements in quantities (i.e. consumed in fixed one-to-one combinations) do not imply that they should also be perfect complements in qualities.[3] There are various ways in which the consumer valuations of the two types of goods could interact in defining the qualities of the combinations these goods form. Therefore, in our model the qualities of the combinations $A_i B_j$, $i = 1, ..., m$, $j = 1, ..., n$ that might be formed by any pair of the available available A-type and B-type goods, are given as a function of the consumer valuations of these goods. Particularly, we assume a simplified version of a CES-form quality-aggregation function:[4]

$$\chi_{ij} = \left\{ [f(A_i)]^\rho + [g(B_j)]^\rho \right\}^{\frac{1}{\rho}} \tag{3}$$

where:
χ_{ij} - the aggregate quality of the combination $A_i B_j$, $i = 1, ..., m; j = 1, ..., n$
$\rho = 1 - \frac{1}{\sigma}$ - coefficient increasing in the elasticity of substitution σ

To reveal the impact of substitutability between the qualities of the two types of goods, we compare the following three cases of interaction between the qualities:

1. linear function: $\chi_{ij} = f(A_i) + g(B_j)$ when $\rho \to 1$ i.e. the two goods are perfect substitutes in qualities

2. simple Cobb-Douglas function: $\chi_{ij} = f(A_i)\, g(B_j)$ when $\rho \to 0$ i.e. the two goods are non-perfect substitutes in qualities

3. Leontief function: $\chi_{ij} = \min\{f(A_i), g(B_j)\}$ when $\rho \to -\infty$ i.e. the two goods are perfect complements in qualities

In compliance with the classical approach of Mussa and Rosen (1978), we assume that consumers differ by taste and model their preferences by the following utility function:

$$u(\chi_{ij}, \theta) = \theta\chi_{ij} - p(\chi_{ij}) = \theta\chi_{ij} - p_i^A - p_j^B \tag{4}$$

where:
θ - taste variable by which consumers are identified, $\theta \sim U(\underline{\theta}, \overline{\theta}), 0 < \underline{\theta} < \overline{\theta}$
$p(\chi_{ij}) = p_i^A + p_j^B$ - total consumer expenditure for the combination of goods A_i and B_j
$p_i^A = p[f(A_i)]$ - price of good A_i as a function of its quality $f(A_i)$, $i = 1, ..., m$
$p_j^B = p[g(B_j)]$ - price of good B_j as a function of its quality $g(B_j)$, $j = 1, ..., n$

The demand side of the market is represented by a continuum of consumers who make individual and mutually exclusive purchases (i.e. buy one combination or do not buy any good at all) from all the possible $m \cdot n$ combinations of the available goods of the two types A and B. We follow the approach of Mussa and Rosen (1978) to identify consumers

[3] We are grateful to Avner Shaked for pointing out this issue in his comments to an earlier draft of the paper.
[4] We are grateful to Vahagn Jerbashian for suggesting CES form of the quality aggregation function for its property to encompass the full spectrum of possible qualitative substitutability between the two complementary goods.

by their taste for quality. The latter is assumed to be uniformly distributed on the support interval $\left[\underline{\theta},\ \overline{\theta}\right], 0 < \underline{\theta} < \overline{\theta}$. These simplification assumptions, common for the models of vertical product differentiation, imply that the demand for a good coincides with the market share of its producer. Thus, they facilitate the representation of the solution of the producer problem.

Likewise Kováč (2007), we assume that each consumer has the same taste for the quality of the both types of goods. Alternatively, it could be argued that the taste variable θ is type-specific. That is, there can be defined a different support interval for each type of good. Then, instead of modelling consumer valuation by a product of a type-irrelevant taste variable with a quality aggregation function, a consumer valuation aggregating function, e.g. $\chi_{ij} = \left\{ \left[\theta_A\ f\left(A_i\right)\right]^{\rho} + \left[\theta_B\ g\left(B_j\right)\right]^{\rho} \right\}^{\frac{1}{\rho}}$, could be directly proposed to replace the minuend in (4). The adoption of such an approach[5] would change the form rather than the intuition behind our results in a sense that the conditions will be defined by means of four $\{\underline{\theta}_A, \overline{\theta}_A, \underline{\theta}_B, \overline{\theta}_B\}$ instead of only two $\{\underline{\theta}, \overline{\theta}\}$ consumer taste endpoint parameters. Furthermore, there is a well-established proof in the literature that consumer taste could be regarded as a measure of the inverse marginal rate of substitution between consumer income and quality (see Tirole, 1988, p.96). Hence, the taste for the quality of a good should be strictly positively correlated with the income of its consumer. Therefore, given that consumer incomes do not vary across the products for which they are spent, we find it reasonable to assume the same for consumer tastes. Finally, there is also a technical argument to assume type-irrelevant consumer taste variable. Namely, the single-dimensional consumer taste space is preferable because it enables the direct comparison of our equilibrium solution with that of the single-market case which we take as a benchmark in our paper.

Together with the similarities, our model has also inevitable divergences from the single-market models. The function in (4) includes two modifications of the original function of Mussa and Rosen (1978) to make it applicable to our model of a vertically differentiated market for combinations of complementary goods. The first modification is prompted by the perfect complementarity of the goods being combined. It implies that consumers could only enjoy combinations containing both types of goods in one-to-one ratio but gain nothing from buying a single good only. We reflect this difference from the single-market model in the utility function of (4) by replacing the single-good quality parameter (χ_i) of Mussa and Rosen (1978) by the combination quality parameter χ_{ij} defined by the function in (3). The second modification we make to the utility function suggested by Mussa and Rosen (1978) is reasoned by another trivial difference from the single-good case. Namely, the total expenditure $p\left(\chi_{ij}\right)$ which consumers incur when purchasing given combination A_iB_j is not set by any particular firm alone but is instead given by the sum of the prices, p_i^A and p_j^B, set by the two firms supplying the goods included in that combination.

To avoid implausible negative utilities, in the single-market models, consumers are

[5] We are grateful to Eugen Kováč and Martin Peitz for pointing us out the adoption of a multi-dimensional consumer taste space as a less restrictive alternative to our approach.

commonly assumed to have an outside option, not to buy any quality available in the market but to consume a free good of inferior non-negative quality. Here, for simplicity and without loss of generality, we assume that the outside option is of zero quality. In addition, since we do not have a single type of good but combinations of paired goods of two different types we introduce positive lower bounds, $\underline{F}$ and $\underline{G}$, on the quality choice of firms producing goods of type A and type B, respectively:

$$\begin{aligned} f\left(A_i\right) \geq \underline{F} > 0 \text{ for } \forall i \in \left\{1, ..., m\right\} \\ g\left(B_j\right) \geq \underline{G} > 0 \text{ for } \forall j \in \left\{1, ..., n\right\} \end{aligned} \tag{5}$$

The inequality conditions in (5) impose minimal quality requirements for a good of given type to be considered a market good. Accordingly, the minimal quality requirement is the same for all producers of a particular type of good.

To establish the producers' decision problem, we next need to derive the demand functions that correspond to the consumer preferences expressed by the utility function in (4). Particularly note that in our model the market space is given by the support interval of the consumer taste distribution. Accordingly, the demand shares of the possible combinations are represented by subintervals within the support interval, enclosed by the so-called marginal taste variables. Each marginal taste variable, generally denoted by θ_{ij/i^*j^*}, corresponds to a taste for quality at which a consumer would be indifferent between buying combination A_iB_j or combination $A_i^*B_j^*$, where A_iB_j is of higher quality than $A_i^*B_j^*$. For easier identification, from now on, we will call it marginal taste variable of combinations $A_i^*B_j^*$ and A_iB_j.

By making use of the utility functional form defined in (4), we could show that the marginal taste variable is given by the ratio between the price difference and the quality difference of the two combinations:

$$\begin{aligned} & u\left(\chi_{ij}, \theta_{ij/i*j*}\right) = u\left(\chi_{i*j*}, \theta_{ij/i*j*}\right) \Rightarrow \theta_{ij/i*j*} = \frac{p\left(\chi_{ij}\right) - p\left(\chi_{i*j*}\right)}{\chi_{ij} - \chi_{i*j*}}, \\ & \text{for any } \ i = 1, ..., m; \ \ j = 1, ..., n; \ \ i^* = 1, ..., m; \ \ j^* = 1, ..., n; \\ & \text{such that } \ \ (i^* \neq i) \ \wedge (j^* \neq j) \end{aligned} \tag{6}$$

where:

θ_{ij/i^*j^*} – marginal taste variable of combinations $A_i^*B_j^*$ and A_iB_j at which consumers are indifferent between buying the one or the other, given that $\chi_{i^*j^*} < \chi_{ij}$

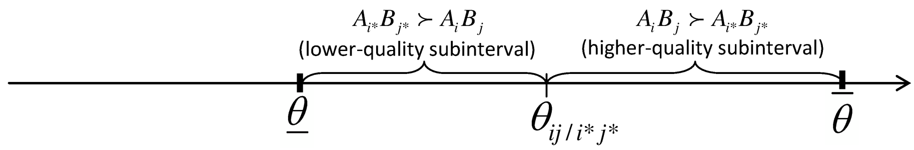
Similarly, we could derive also the marginal taste $\theta_{ij/0}$ at which a consumer would be indifferent between buying combination A_iB_j or not purchasing in the market at all, as follows:

$$u\left(\chi_{ij}, \theta_{ij/0}\right) = u\left(\chi_0 = 0, \theta_{ij/0}\right) \Rightarrow \theta_{ij/0} = \frac{p_i^A + p_j^B}{\chi_{ij}} \tag{7}$$
$$\text{for any } \ i = 1, ..., m; \ \ j = 1, ..., n;$$

where:
$\theta_{ij/0}$ – marginal taste variable of combination A_iB_j and the free outside option O at which consumers are indifferent between buying the former and switching to the latter

As in the single-market case, when a marginal taste is within the range of variation of consumer tastes, it plays the role of a boundary which divides consumers' population into two groups. Or technically speaking, in our model any marginal taste variable belonging to the support interval $[\underline{\theta}, \ \overline{\theta}]$ divides the support interval into two subintervals as shown in figure 1 below. One of the group consists of the consumers with tastes lower than $\theta_{ij/i*j*}$. If asked at the given prices to only choose between the two combinations, A_iB_j and $A_{i^*}B_{j^*}$, they will strictly prefer the latter to the former. Therefore, we call the subinterval of the tastes of the consumers belonging to this group, the lower-quality subinterval of $\theta_{ij/i*j*}$. On the contrary, the other group of consumers with tastes larger than $\theta_{ij/i*j*}$, will strictly prefer A_iB_j to $A_{i^*}B_{j^*}$. In turn, we call the subinterval of the tastes of the consumers belonging to this second group, the higher-quality subinterval of $\theta_{ij/i*j*}$.

Figure 1: Division of the support interval $[\underline{\theta}, \ \overline{\theta}]$ into the lower-quality and upper-quality subintervals of $\theta_{ij/i*j*}$.



Alternatively, if the marginal taste does not belong to the range of variation of consumer tastes, at the given prices all consumers will prefer either the one or the other combination. Nobody will be indifferent. In technical terms, if a marginal taste variable $\theta_{ij/i*j*}$ is smaller than the lower endpoint $\underline{\theta}$ of the support interval, at the given prices its higher-quality subinterval will coincide with the support interval whereas its lower-quality subinterval will be empty. Similarly, if a marginal taste variable $\theta_{ij/i*j*}$ is larger than the upper endpoint $\overline{\theta}$ of the support interval of consumer taste distribution, at the given prices its lower-quality subinterval will coincide with the support interval whereas its higher-quality subinterval will be empty.

In order to define generally the demand for any combination A_iB_j, $i = 1, ..., m$; $j = 1, ..., n$, it is useful also to categorize its marginal taste variables based on whether A_iB_j is demanded by their lower- or higher-quality intervals. The first category includes the marginal taste variables in the indices of which the combination A_iB_j is positioned after the slash. These are the marginal taste variables whose lower-quality subintervals represent the consumers who at the given prices prefer A_iB_j to other higher-quality combinations. Therefore, from now on we will refer to them by calling them lower-quality marginal taste variables of combination A_iB_j. In a similar manner, we will call higher-quality marginal taste variables of A_iB_j the marginal taste variables in the indices of which the combination A_iB_j is positioned before the slash. That is, the higher-quality subinterval of each higher-quality marginal taste variable of A_iB_j represents the consumers who at the given prices prefer A_iB_j to another lower-quality combination.

The demand for any combination $A_iB_j, i = 1, ..., m; j = 1, ..., n$, is given by the intersection of all its higher- and lower-quality subintervals and the support interval of consumers' taste distribution. In particular, it is equal to the difference between the smallest higher-quality marginal taste variable $\overline{\theta}_{ij}$ and the largest lower-quality marginal taste variable $\underline{\theta}_{ij}$ of any particular combination A_iB_j provided that the difference is positive. Otherwise, if the difference is non-positive, the demand for the combination is zero:

$$D_{ij} = max\{(\overline{\theta}_{ij} - \underline{\theta}_{ij}),\ 0\}, i = 1, 2,\ \dots, m;\ j = 1, 2,\ \dots, n; \tag{8}$$

where for any $i = 1, 2,\ \dots, m$ and $j = 1, 2,\ \dots, n$ we have:
$\overline{\theta}_{ij}$ – the smallest higher-quality marginal taste variable of combination A_iB_j, i.e. $\overline{\theta}_{ij} = min\{\theta_{\psi(1)/ij},\ \dots,\ \theta_{\psi(r_{ij}-1)/ij},\ \overline{\theta}\}$ so that:
r_{ij} – quality rank of combination A_iB_j; $r_{ij} \in \{1, 2, \dots, m \cdot n\}$
$\psi(r_{ij})$ – inverse rank-translating function which takes the quality rank of any combination A_iB_j as an argument and returns the index ij formed by the ordered pair of quality ranks of the goods A_i and B_j of which the combination is formed, see table 1 below
$\underline{\theta}_{ij}$ – the largest lower-quality marginal taste variable of combination A_iB_j, i.e. $\underline{\theta}_{ij} = max\{\theta_{ij/\psi(r_{ij}+1)},\ \dots,\ \theta_{ij/\psi(m \cdot n)},\ \theta_{ij/0},\ \underline{\theta}\}$

Since the best-ranked combination A_1B_1 does not have a higher-quality marginal taste variable ($r_{11} = 1$), the minuend of the difference term in its demand function is given by the right endpoint $\overline{\theta}$ of the support interval of the consumer taste distribution. Similarly, if the smallest higher-quality marginal taste variable of a combination exceeds the right endpoint $\overline{\theta}$, the latter replaces it as a minuend of the difference term in the demand function of the combination. Also, if the largest low-quality marginal taste variable of a combination is smaller than the left endpoint $\underline{\theta}$ of the support interval of the consumer taste distribution, the latter replaces it as a subtrahend of the difference term in the demand function of the combination.

In the current paper, we explore two particular market settings. First, we define the equilibrium in a market with single entrant of type A ($M = 1$) that is assumed to have higher quality than any of the N potential entrants of type B ($f(A_1) > g(B_1)$). Second, we solve for an equilibrium where the mixed-quality combinations are effectively excluded from a market with two actual entrants of type A and type B, respectively.

Below, we provide graphical illustration how the intersection of the higher- and lower-quality subintervals give the equilibrium demands for the best ranked combination A_1B_1 in the first setting (Figure 2) and for the worst-ranked combination A_2B_2 in the second setting (Figure 3).

The two equilibrium outcomes depicted in figure 2 and figure 3 represent two mechanisms in which a combination might be efficiently excluded from the market. In figure 2 we have an equilibrium outcome where the price of one of the goods which is present in all the combinations is so high relative to the qualities of the ones excluded that consumers prefer to switch to the free outside option rather than to buy any of these low-ranked combinations. In figure 3, the qualities of the excluded combinations do not differ sufficiently from the quality of the worst-ranked combination for the suppliers of their goods to prefer all the combinations sold. Accordingly, at the optimal prices that firms choose the less-differentiated combinations are effectively excluded in favor of the lower-quality, but more differentiated, worst-ranked combination. Neither of the two exclusionary outcomes can occur at equilibrium in a single-market setting.

There is also a third mechanism for excluding a good, which Shaked and Sutton (1982) prove to be working in a single-good market. It requires the lowest consumer identification variable, in our model $\underline{\theta}$, to be sufficiently low so that even when a good is charged zero price, its lower-quality taste-variable is below $\underline{\theta}$. In the current paper, this third mechanism plays a role in preventing the entry of a third firm of type A and/or B. It enters the solution by defining the necessary condition for having only two actual entrants in the extended version of the second setting allowing for more than two potential entrants of a type. We do not provide graphical illustration of this outcome below because it does not directly refer to the definition of the demand expression in (8).

In the first setting since we have a single potential entrant of type A whose good's quality exceeds the quality of any other good of type B, for any of the three special cases of the aggregation function in (3), the ranking of possible combinations follows the ranking of the goods of type B, as shown in table 1 below:

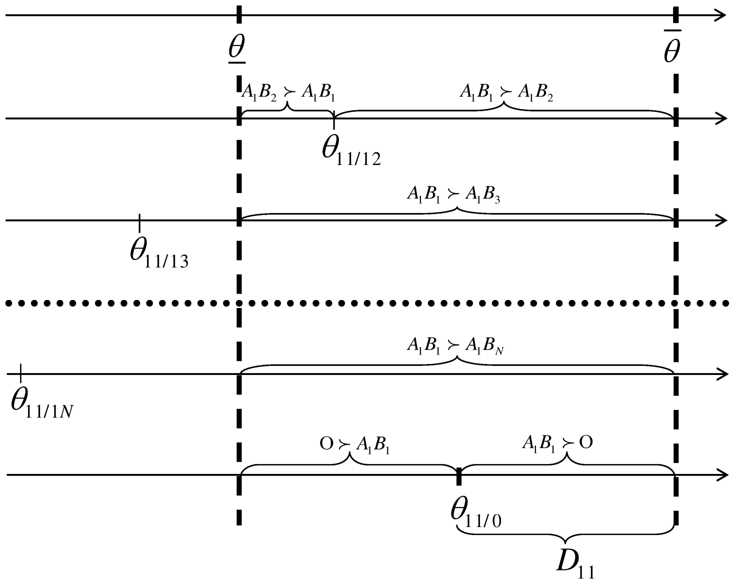
Table 1: Ranking of combinations given 1 entrant of type A and n entrants of type B

rank (r_{ij})	**combination** (A_iB_j)	**index** ($\psi(r_{ij})$)
1	A_1B_1	11
2	A_1B_2	12
3	A_1B_3	21
...	...	...
n	A_1B_n	1n

Then, as shown in figure 2 below, at equilibrium only two marginal taste variables, $\theta_{11/12}$ and $\theta_{11/0}$, have values that exceed the left endpoint $\underline{\theta}$ of the support interval of the consumer taste distribution.

The value of the marginal taste variable $\theta_{11/0}$ is so high that it exceeds not only $\underline{\theta}$ but

Figure 2: Demand D_{11} for combination A_1B_1 defined by the intersection of the subintervals of its marginal taste variables and the support interval of the consumer-taste distribution given 1 potential entrant of type A and N potential entrants of type B.



also the marginal taste variable $\theta_{11/12}$. As a result, there are consumers with tastes lower than $\theta_{11/0}$ and higher than $\theta_{11/12}$ who prefer the free outside option to combination A_1B_1 and by transitivity to combination A_1B_2, as well. Accordingly, the consumers with tastes lower than $\theta_{11/12}$ will also by transitivity prefer the outside option to A_1B_2. Thus, the equilibrium outcome presented in figure 2 implies that all consumers prefer to buy either the best combination A_1B_1 or the free outside option which makes good B_2 unsalable. By induction we could show that this would be the case with any other B-type producer whose good is not going to be ranked the best after entry. Therefore, at any positive spread of consumer tastes, provided that there is a single entrant of type A, A_1B_1 will be the only salable combination with demand for it given by the difference between $\overline{\theta}$ and $\theta_{11/0}$. The right endpoint $\overline{\theta}$ of the support interval takes the place of the minimal higher-quality marginal variable $\theta_{i-j-/ij}$ in (8) because A_1B_1 is the best-ranked combination and as such cannot have a higher-quality marginal variable. Accordingly, $\theta_{11/0}$ is the largest lower-quality marginal variable of A_1B_1 and as such it takes the place of the subtrahend in the general demand function (8) for $i = j = 1$.

Such a market where not all consumers are served by the incumbent firms but some of the low-taste consumers instead switch to the outside option is said to be not covered by these firms. Accordingly, the market coveredness and the number of entrants with positive market shares are commonly recognized in the existing literature as defining characteristics of the equilibrium outcome in any market with product differentiation. Since in our setting we have only single-product firms neither of which alone is able to define the amount $p(\chi_{ij})$ spent for any particular combination of goods A_iB_j, in what follows we prefer to talk not about firms but about combinations covering the market.

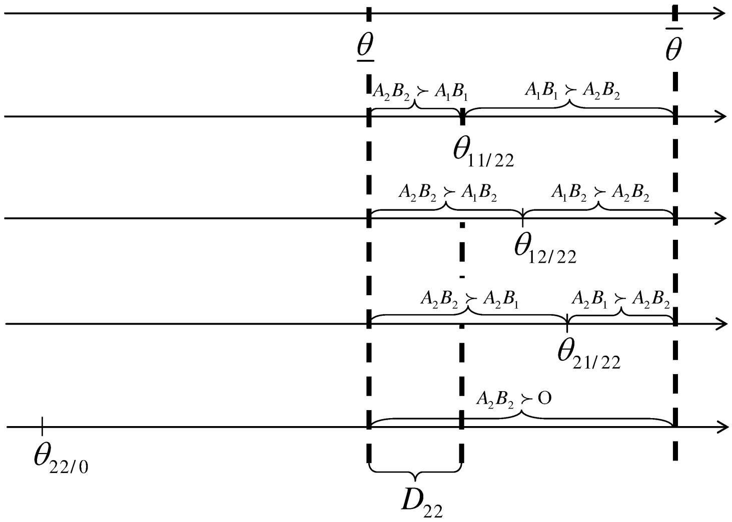
In the second setting that we consider, there are two actual entrants of each type offering goods of distinct qualities, so there could be two different rankings of their combinations as follows:

Table 2: Possible rankings of the combinations given 2 entrants of both types A and B

rank	**Ranking 1**	**Ranking 2**
1	A_1B_1	A_1B_1
2	A_1B_2	A_2B_1
3	A_2B_1	A_1B_2
4	A_2B_2	A_2B_2

In figure 3 below, a particular equilibrium outcome is depicted for the case when ranking 1 holds.

Figure 3: Demand D_{22} for combination A_2B_2 defined by the intersection of the subintervals of its marginal taste variables and the support interval of the consumer-taste distribution given 2 entrants of each type, A and B.



As we will show in the next section, under certain conditions sufficient for having the equilibrium outcome in figure 3 the outcome is the same at the both rankings 1 and 2. Namely, $\theta_{11/22}$ is of lower value than $\theta_{11/12}$ and $\theta_{11/21}$. Accordingly, there are consumers with tastes larger than $\theta_{11/22}$ and smaller than $\theta_{11/12}$ and $\theta_{11/21}$ who at the equilibrium prices prefer to buy A_1B_1 to A_2B_2, and A_2B_2 to A_1B_2 and A_2B_1, respectively. By transitivity, this implies that all consumers with tastes larger than $\theta_{11/22}$ buy A_1B_1 but not A_1B_2 or A_2B_1. By the same reasoning, all consumers with tastes smaller than $\theta_{11/22}$ buy A_2B_2 but not A_1B_2 or A_2B_1. Thus, only the best- and the worst-ranked combinations are salable at equilibrium.

Based on the consumers' demand for the possible combinations of the available goods of each type, producers solve a decision problem which can be represented by the following 3-stage game:

Stage 1: Firms simultaneously decide to enter the market or not.

There are M potential entrants of type A and N potential entrants of type B. There are no multi-product firms. Accordingly, each entrant faces different technological frontier which is a restriction from above on the characteristics of the best good that it is capable to produce. This is important for the entry decision of each potential entrant because upon entry besides the other features the restriction from above will constrain also the mutually-agreed mix of characteristics by which consumers value the type of good the entrant offers. Therefore, we introduce it in the model as an individual upper bound on the quality choice of each firm which is already known by the potential entrants before the entry. From now on we will call this upper bound maximal quality constraint.

Since all consumers agree on the ranking of the characteristics of the goods of the same type, ranking the potential entrants in descending order of their maximal quality constraints should not vary among the consumers. Hence, the rank of the maximal quality constraint of a potential entrant is a unique identifier that allows us to distinguish it from the others. Indeed, it represents the rank of the best potential product a firm could produce in a kind of a virtual pre-market competition with the other potential entrants of the same type. Thus, the rank of a potential entrant's maximal quality constraint plays the same role as firm's brand is meant to play in the real world. It sends a signal to the consumers (as well as to the other external stakeholders of the potential entrant) what is the best quality that a firm is capable to produce. Therefore, below we rank in descending order the maximal quality constraints of the potential entrants of type A and B and denote them by $\bar{F}_k,\ k = 1,\ \dots, M$ and $\bar{G}_l,\ l = 1,\ \dots, N$, respectively:

$$\bar{F}_1 > \bar{F}_2 > \bar{F}_3 > \ \dots\ > \bar{F}_M \tag{9}$$

$$\overline{G}_1 > \overline{G}_2 > \overline{G}_3 > \ \dots\ > \overline{G}_N \tag{10}$$

Stage 2: Entrants choose simultaneously the qualities of their goods. Let m entrants of type A and n entrants of type B be in the market after the first stage, $m \in \{1, \dots, M\}$, $n \in \{1, \dots, N\}$. We identify actual entrants in the same way as the potential entrants. Each actual entrant of type A is associated with the rank i of its maximal quality constraint $\bar{F}_{k_i},\ i = 1, \dots, m$ within the subsequence of sequence (9) which is formed by arranging in descending order the maximal quality constraints of the actual entrants of type A, as follows:

$$\bar{F}_{k_1} > \bar{F}_{k_2} > \bar{F}_{k_3} > \ \dots\ > \bar{F}_{k_m} \tag{11}$$

Similarly, each actual entrant of type B is associated with the rank j of its maximal quality constraints $\bar{G}_{l_j},\ j = 1,\ \dots, n$ within the subsequence of sequence (10) which is formed by arranging in descending order the maximal quality constraints of the actual entrants of type B, as follows:

$$\bar{G}_{l_1} > \bar{G}_{l_2} > \bar{G}_{l_3} > \ \dots\ > \bar{G}_{l_n} \tag{12}$$

To make clear the transition from potential entrants' ranking to actual entrants' ranking, let us imagine for example a situation where the first two best-ranked potential entrants of type A decide not to enter in the first stage of the game but the third best-ranked potential entrant decides to enter the market. Then, the third best-ranked potential entrant will in fact have the best-ranked maximal quality constraint from all the actual market entrants of type A so that in stage 2 its constraint from above will have rank 1, that is $\bar{F}_{k_1} = \bar{F}_3$. As we will show later this will never be the case at equilibrium. In fact, at equilibrium the m actual entrants are the m best-ranked potential entrants. For neither of them it can be optimal to stay out of the market given that at the same time market entry is an optimal choice for any lower-ranked potential entrant. Similar example and comment about the equilibrium outcome after the entry stage can be made also for the entrants of type B. That is, the rankings in (11) and (12) characterize only the transition to the equilibrium outcome but not the equilibrium itself. The equilibria established in the next section are therefore definable by solely using the notation for the maximal quality constraints of the potential entrants in (9) and (10).

After observing how many firms of each type have entered the market in the first stage, every actual entrant i of type A chooses the quality of its good $f(A_i)$ from within the interval $\left[\underline{F}, \bar{F}_{k_i}\right]$, while every actual entrant j of type B chooses the quality of its good $g(B_j)$ from within the interval $\left[\underline{G}, \bar{G}_{l_j}\right]$. Entrants make their quality choices simultaneously. The chosen qualities represent the same mix of characteristics of the goods of each type on the ranked value of which consumers are assumed to mutually agree according to (1) and (2).

Stage 3: Firms compete in prices.

After making their quality choices in the second stage, firms observe the available qualities in the market and choose simultaneously the prices of their goods. Possibilities for product extension mergers, collusions and tying practices imposing pre-selected bundles on consumers are precluded in the model.

The payoffs of the firms are given by their profits. For simplicity, production costs are assumed to be zero for all the producers of both types so that for each firm profit coincides with revenue. The demand share of a good is given by the sum of the demand shares of the combinations in which it takes part. Accordingly, dependent on the type of its good the decision of a market entrant at the first and second stage, could be jointly represented by one of the following profit-maximization problems:

$$\max_{f(A_i), p_i^A} \Pi_i^A = R_i^A = p_i^A D_i^A = p_i^A \sum_{j=1}^{n} D_{ij},\ i = 1, ..., m \tag{13}$$

$$\max_{g(B_j), p_j^B} \Pi_j^B = R_j^B = p_j^B D_j^B = p_j^B \sum_{i=1}^{m} D_{ij},\ j = 1, ..., n \tag{14}$$

where:

Π_i^A - profit of the producer of good of type A with quality rank i

Π_j^B - profit of the producer of good of type B with quality rank j

R_i^A - revenue of the producer of good of type A with quality rank i

R_j^B - revenue of the producer of good of type B with quality rank j
D_i^A - demand share of good of type A with quality rank i
D_j^B - demand share of good of type B with quality rank j

If a firm decides not to enter the market in the first stage, its profit is zero.

We solve the model first with exogenously set number of entrants. We start with a setting of the model where there are a single entrant of type A and n potential entrants of type B. We assume that the consumer taste distribution satisfies the condition of Shaked and Sutton (1982) for having at least two entrants with positive market shares at a single-market equilibrium. Our aim is to show that still only the firm offering the best-ranked good of type B could have positive market share at the equilibrium with only one entrant of type A. Then, we continue with analyzing a setting with two entrants of type A and two entrants of type B to define the conditions expressed in terms of the constraints on the quality choices of the firms for having the mixed-quality combinations unsalable at a covered-market equilibrium. Again the consumer taste distribution is assumed to satisfy the single-market condition of Shaked and Sutton (1982) for having exactly two entrants with positive market shares. Finally, we define sufficient conditions for having at most two entrants of a type at equilibrium and show that these conditions are stricter than the single-market conditions of Shaked and Sutton (1982). Accordingly, we establish an equilibrium at which exactly two firms of each type enter the market but only the best and the worst combinations in the market have positive market shares and cover the market. The results are formally presented and discussed in the next section.

3 Equilibrium Solution

In this section we discuss the results of the solution of our model. Here we present only the intuition behind the equilibrium outcome while rigorous solution is provided in the appendix. We are interested in exploring the conditions for having a covered-duopoly market where only the best and worst combinations have positive market shares. However, we reach to it gradually.

First, we establish the equilibrium solution for the case with only a single entrant of type A who faces higher maximal quality constraint than the n entrants of type B. Accordingly, for any of the three special cases of the aggregation function in (3), the ranking of possible combinations follows the ranking of the goods of type B as shown in the previous section (table 1). We show that the variability of the consumer identification variable (i.e. the taste variable in our model) should satisfy the same condition as in Shaked and Sutton (1982) for having at most two entrants of type B with positive market shares. This is formally stated in proposition 1 below.

Proposition 1. *Given that the number of entrants of type A is restricted to one, let the parametric endpoints $\underline{\theta}$ and $\overline{\theta}$ of the support interval, on which consumers' tastes are assumed to be distributed, satisfy the following condition:*

$$\frac{\overline{\theta}}{4} < \underline{\theta} \tag{15}$$

Then, of any n potential entrants of type B at most two will have positive market share at equilibrium.

Proof. See section A of the appendix □

Distinct from the single-market equilibrium outcome, at certain conditions the optimal solution of the particular case with single entrant offering a complementary good of type A implies not two but only one entrant of type B with positive market share. Furthermore, the result is independent on the distribution of the taste variable. As shown in figure 2, A_1B_1 is the only possible combination demanded by consumers at the optimal price chosen by the single entrant of type A. By being the sole entrant of type A, the supplier of good A_1 is in a similar position to that of a classical monopolist. It maximizes its revenue by setting a price at which only the consumers with elastic demand would buy its good. However, when the best-ranked good of type B is sufficiently differentiated from the goods of the rest of the potential entrants, all the consumers with elastic demand buy A_1 only at equilibrium. That is, the taste at which consumer demand for A_1B_1 is exactly unit-elastic is actually the one at which consumers would be indifferent between buying the best-ranked combination or switching to the outside option. Accordingly, if there are consumers with lower tastes, they will strictly prefer the outside option. That is, the market will not be covered at equilibrium. The result is established in proposition 2 below.

Proposition 2. *Given that the number of potential entrants of type A is restricted to one and the maximal-quality constraints of the two-best-ranked goods of type B are related as follows:*

$$\bar{F}_1 > \bar{G}_1 > \frac{9}{4}\bar{G}_2 \tag{16}$$

of any n potential entrants of type B exactly one will have positive market share at equilibrium. The result holds for any $\underline{\theta} < \bar{\theta}$ and for $\sigma \to \{0, 1\}$. Furthermore, the market will not be covered as long as the following condition holds:[6]

$$\underline{\theta} < \frac{\overline{\theta}}{2} \tag{17}$$

Proof. See section B of the appendix □

This situation resembles a standard case in the literature on applying product tying as a market power leverage device where the producers of the two types of goods are assumed to offer them in different markets. As a result, the firm operating as a monopolist in one

[6] In a single-good model, condition (17) is sufficient for at least two goods of distinct qualities to have positive market share. Together with condition (15), it ensures covered duopoly at equilibrium, see (Shaked and Sutton, 1982, p.7).

of the markets is in a position to decide whether to exclude the lower-quality producer in the other duopoly market (Whinston, 1990, p. 841). Therefore, the monopoly market is usually designated as core market while the competitive market is designated as adjacent market whereas the good sold on the core market is called bottleneck good (Rey and Tirole, 2007, p. 2183). Since in our setup firms make decisions independently, we could consider the entrant of type A as operating in a separate market, let us call it market A, from the market in which the entrants of type B operate, let us call it market B. Accordingly, good A_1 plays the role of a bottleneck good. Distinct from the situation described in (Whinston, 1990), however, the market-power leverage outcome is achieved in our solution without the entrant in the core market A to be necessary to tie to its good the good of any of the potential entrants in the adjacent market B. Indeed, the consumers are free to buy the bottleneck good A_1 in combination with the good of any of the n entrants of type B. However, the consumers, who value A_1 sufficiently high to buy it at its monopoly price, are too demanding to compromise with the quality of the best-ranked good of type B which discourages the lower-ranked firms of type B from entering the market.

The result established in proposition 2 implies that the presence of a complementary good of superior quality offered by a monopolist in market A might change the equilibrium outcome in market B from covered duopoly into non-covered monopoly. The condition for having such an outcome at equilibrium is the best-ranked good in market B to be sufficiently differentiated from the rest B-type goods as required by the second inequality in (16). Furthermore, in section B of the appendix we explore also the possible equilibrium solutions if the best-ranked good of type B is not so differentiated. We show that a non-covered equilibrium with more than one actual entrant in market B cannot exist. The single entrant in market A will always apply exclusionary pricing given non-covered market. An outcome with more than one entrant could occur only at a covered-market equilibrium. Still we could have at most three entrants because the existence of a covered-market equilibrium requires consumer taste spread to be restricted from below $\left(\frac{\overline{\theta}}{7} < \underline{\theta}\right)$. Otherwise, the producer of type A again prefers to charge exclusionary price of its good. Accordingly, for having exactly three entrants at equilibrium also the condition in (15) needs to be relaxed $\left(\underline{\theta} < \frac{\overline{\theta}}{4}\right)$.

In the second setting for which we provide a solution of our model there are two potential entrants of type A and two potential entrants of type B. By analyzing it, we move towards answering the key question in the present paper. Namely, we are interested to define what are the conditions under which an equilibrium exists at which only the most differentiated possible combinations of the available goods are actually sold.

In contrast to the first setting, here, there is no asymmetry between the two types of goods in terms of the number of potential entrants that offer them. We have two potential entrants of each type, A or B. Therefore, the ranking of the possible combinations cannot be deduced straight from the difference in the qualities of the best-ranked goods only. Indeed, as it is shown in table 2 of the previous section, when there are two potential entrants of each type, their goods have four possible combinations for which two possible rankings exist. For any of the three special cases of the aggregation function in (3), combination A_1B_1 formed by the best goods of both types has the lowest rank 1 whereas the combination A_2B_2 formed by the worst goods of both types has the highest rank 4. The only uncertainty is which of the two mixed-quality combinations is better, A_1B_2 or A_2B_1. The determinant of the preference relation between the two combinations is given by the relation between the qualities of type A and type B. The particular condition, however, depends on the elasticity of substitution σ between the qualities of the two types which is present through the parameter ρ in the quality-accumulation function in (3). The rigorous expressions of the conditions for each relation in the three special cases of substitutability between the qualities of the two types of goods are presented in part C of the appendix. Proposition 3 below establishes general sufficient condition for having ranking 1 in table 2 at any of the three values which ρ is assumed to approach.

Proposition 3. *Given two entrants of each type in the market and $\sigma \to \{-\infty, 0, 1\}$, the combinations formed by the better good of type A would be of higher rank than the combinations formed by the worse good of type A if the following relation between the qualities of the available goods holds:*

$$f(A_1) > g(B_1) > g(B_2) > f(A_2) \tag{18}$$

Proof. See section C of the appendix □

Since ranking 2 is a mirror image of ranking 1, a general sufficient condition for it could also be derived. It is given by trading the places of $f(A_i)$ and $g(B_j)$ for $i = j$ in (18):

$$g(B_1) > f(A_1) > f(A_2) > g(B_2) \tag{19}$$

The quality relations in (18) and (19) imply that the type of good with greater differentiation is the one whose quality matters more for the ranking of the combinations. That is, from the two mixed-quality combinations the one that has the better-ranked good of the more differentiated type is also of higher rank.

No matter which of the two mixed-quality combinations is of higher rank, we cannot have both mixed-quality combinations salable at a covered-market equilibrium if the quality difference between the combination with rank 2 and the best combination exceeds that between the combination with rank 3 and the worst combination:

$$\begin{aligned} &\chi_{11} - \chi_{12} > \chi_{21} - \chi_{22}, \text{ if ranking 1 or,} \\ &\chi_{11} - \chi_{21} > \chi_{12} - \chi_{22}, \text{ if ranking 2} \end{aligned} \tag{20}$$

When the condition in (20) holds, the denominator of the marginal taste variable of the former pair of combinations is larger than that of the latter pair. At the same time, the general expression in (7) implies that the two marginal taste variables have the same numerator given by the difference of the prices of the goods of the less differentiated type.

Hence, the marginal taste variable of the higher-ranked pair of combinations is smaller than that of the lower-ranked pair of combinations:

$$\begin{aligned}
\theta_{11/12} &= \frac{p_1^B - p_2^B}{\chi_{11} - \chi_{12}} < \frac{p_1^B - p_2^B}{\chi_{21} - \chi_{22}} = \theta_{21/22}, \text{ if ranking 1 or,} \\
\theta_{11/21} &= \frac{p_1^A - p_2^A}{\chi_{11} - \chi_{21}} < \frac{p_1^A - p_2^A}{\chi_{12} - \chi_{22}} = \theta_{12/22}, \text{ if ranking 2}
\end{aligned} \tag{21}$$

The inequalities in (21) imply that we cannot have both the former marginal taste variable $\theta_{11/12}$ (resp. $\theta_{11/21}$) larger and the latter marginal taste variable $\theta_{21/22}$ (resp. $\theta_{12/22}$) smaller than the marginal taste of the two mixed-quality combinations $\theta_{12/21}$ (resp. $\theta_{21/12}$). There is either only one of the combinations salable at equilibrium or neither of the two.

To understand the intuition behind the possible equilibrium outcomes when the rank-related conditions in (20) hold, note that when the worst-ranked combination A_2B_2 is of sufficiently higher quality than the free outside option to cover the market, the combination of rank 3 is the least differentiated from its neighbors by rank. Accordingly, the suppliers of the goods in that combination are subjected to strong competitive pressure to decrease their prices in order to make the combination salable. However, the firm offering the lower-ranked good of the more differentiated type in the combination of rank 3 prefers to keep its price high because the sales of the worst combination, in which its good is also present, are not threatened by the outside option. Similarly, the firm offering the higher-ranked good of the less differentiated type is better-off of selling its good to the highest-taste consumers who buy it as a part of the best-ranked combination. Indeed, the resulting additional revenue compensates for the lost sales of the good as a part of the mixed-quality combination of rank 3.

The competitive pressure experienced by the suppliers of the goods in combination of rank 2 is not that strong as that faced by the firms supplying the goods in combination of rank 3. On the one hand, the inequalities in (20) ensure its higher differentiation from the best-ranked combination compared to the differentiation of the combination of rank 3 from the worst-ranked combination. On the other hand, the good of the less differentiated type in the combination is of lower rank. So, it is alternatively part of the worst-ranked combination sold to the lowest-taste consumers. Therefore, the corresponding revenue would compensate for the lost sales of the combination of rank 2 only as long as it is not much similar by quality to the best-ranked combination rather than to the worst-ranked one:

$$\begin{aligned}
&\chi_{11} - \chi_{12} > \chi_{12} - \chi_{22}, \text{ if ranking 1 or,} \\
&\chi_{11} - \chi_{21} > \chi_{21} - \chi_{22}, \text{ if ranking 2}
\end{aligned} \tag{22}$$

The condition in (22) is necessary and sufficient for having both mixed-quality combinations unsalable at a covered market equilibrium. To establish this equilibrium, however, the condition on the combination qualities in (22) as well as the condition for having covered market need to be translated into a condition on the constraints of the good quality choices of the market entrants of each type. The result is formally presented in proposition 4 below.

Proposition 4. *Given the distribution of consumer's taste according to the conditions in (15) and (17), let two firms of each type enter the market. Then, if whichever of the following alternative relations characterize their quality-choice ranges:*

$$\frac{3}{2}\frac{\underline{F}}{\bar{F}_1}\underline{G} > \bar{F}_1 > \bar{G}_1 > \frac{6}{5}\underline{G} > \frac{6}{5}\underline{F} \tag{23}$$

$$\frac{3}{2}\frac{\underline{G}}{\bar{G}_1}\underline{F} > \bar{G}_1 > \bar{F}_1 > \frac{6}{5}\underline{F} > \frac{6}{5}\underline{G} \tag{24}$$

for $\sigma \to \{0, 1\}$ there exist the following subgame equilibria at the last two stages:

1. *at the quality-choice stage the maximum differentiation outcome prevails: see expressions (D6) and (D7) in the appendix*

2. *at the pricing stage the optimal prices are given by expressions (D1) and (D2) in the appendix.*

Proof. See section D the appendix □

There are three moments in proposition 4 to stress on.

First, the condition in (22) for having the both mixed-quality combinations unsalable holds only if the two goods are not perfectly substitutable in quality. That is, the negative impact of the quality of the lower-ranked good on the aggregate quality of any of the two mixed-quality combinations should not be perfectly compensable by the respective positive effect of the quality of the higher-ranked good. Otherwise, the quality of these combination with rank 2 will be much similar to the quality of A_1B_1 than to the quality of A_2B_2. So, there will be positive demand for it at the equilibrium prices. In the other two forms of the quality aggregation function we consider, namely, non-perfectly substitutable ($\sigma \to 1$) and perfectly complementary ($\sigma \to 0$) qualities, for having the condition in (22) satisfied, it is sufficient the quality of the higher-ranked good of the less differentiated type to exceed that of the lower-ranked one by at least six-fifths.

Second, if there is only demand for the best-ranked and the worst-ranked combinations, the optimal quality choice of the supliers of each of the two types of goods is to maximize the difference from one another. That is, the firm with higher maximal quality constraint of its type will choose it whereas the other firm offering good of the same type will choose the minimal quality constraint. Thus, by setting the range marked by the higher maximal quality constraint and the minimal quality constraint of a type to be a subinterval of the range marked by the corresponding quality constraints of the other type, we will have the latter type being the more differentiated at the equilibrium established in proposition 4. In addition, we require the higher maximal quality constraint of the less differentiated type to exceed the minimal constraint by at least six-fifths to ensure the validity of the sufficient condition for (22) to hold as discussed in the previous paragraph.

Third, as the equilibrium established in proposition 2 for the setting with a single entrant of type A, the equilibrium with two entrants of type A and two entrants of type B in proposition 4 is defined for distribution of consumer identification variable that satisfies the conditions in (15) and (17). They are the ones proven by Shaked and Sutton (1982) to be necessary and sufficient for exactly two entrants with positive market shares to cover a single market at equilibrium. As it was discussed above, for the market to be covered at this setting, the quality of the worst-ranked combination needs to be sufficiently larger than the quality of the outside option, respectively not much smaller than the best-ranked combination. This is ensured by the requirement in (23) and (24) the maximal quality constraint of the higher-ranked good of the more differentiated type to be exceeded by at least one and half times the minimal quality constraint of the less differentiated type of goods.

Finally, we establish the conditions for having exactly two entrants of a type at the entry-stage subgame equilibrium at a setting where there are more than two potential entrants. The key question is if two goods of each type could fit the market whether a third good of any of the two types cannot enter and have positive market share at equilibrium. In this sense, note that the optimal prices are negatively related to the number of market entrants. The more entrants the stronger is the price competition between them. Therefore, if we can establish a condition at which the entry of just a third firm is effectively deterred because it faces too low best-response prices from its competitors, the condition will imply that these prices will be even lower in case of more than three entrants. The effective deterrence of a third entrant again depends on the relations between the qualities of the two types of goods and the corresponding ranking of the combinations they form. The very equilibrium at which a potential third entrant is effectively excluded from the market is established in proposition 5 below.

Proposition 5. *For $\sigma \to \{0,1\}$, let the consumer taste distribution be characterized by the following inequality condition:*

$$\frac{4\left(\chi_{11}-\chi_{22}\right)}{6\chi_{11}-\chi_{22}}\bar{\theta} \leq \underline{\theta} < \frac{2\bar{\theta}}{3} \tag{25}$$

Then, if the quality-choice ranges of the three potential entrants with highest maximal quality constraints satisfy whichever of the following alternative relations:

$$\bar{F}_1 > \bar{G}_1 > 2\bar{G}_3 > 2\ max\{\bar{F}_3, \underline{G}\} > 2\ min\{\bar{F}_3, \underline{G}\} > 2\underline{F} \tag{26}$$

$$\bar{G}_1 > \bar{F}_1 > 2\bar{F}_3 > 2\ max\{\bar{G}_3, \underline{F}\} > 2\ min\{\bar{G}_3, \underline{F}\} > 2\underline{G} \tag{27}$$

there exists subgame-perfect equilibrium established by the solution of the three-stage game below:

1. *Only the two firms with the highest maximal quality constraints of each type enter the market, the rest do not enter.*

2. *The optimal quality choices of the two entrants imply maximum product differentiation (see expressions (D6) and (D7) in the appendix)*

3. *The optimal prices are given by expressions (D1) and (D2) in the appendix.*

Proof. See section E of the appendix □

The effective market exclusion of a third entrant of whichever of the two types requires wider range of the quality choices $\left(\bar{G}_1 > 2\underline{G}\right)$ or $\left(\bar{F}_1 > 2\underline{F}\right)$ than is sufficient for having the subgame equilibria with exogenously set number of entrants in proposition 4. As a result, the condition for covered market in (25) is more relaxed than in (15) and stricter than in (17). It restricts the lowest consumer taste from below instead of the higher maximal constraints from above. The optimal quality and price choices remain the same as in proposition 4.

4 Conclusion

In this paper we introduce a stylized model of price competition in two adjacent vertically differentiated markets for complementary products manufactured and offered by different firms. The setting analyzed here differs from the case considered in the classical literature on vertical differentiation where goods are assumed to be offered independently in a single market.

We show that when the two types of goods combined are complements, thus consumed in a fixed one-to-one ratio, the producers of the high-quality goods of each type might find it optimal under certain conditions to charge prices at which the combinations they could form with the low-quality goods of the other type are unsalable. As a result, if there is only a single entrant in one of the markets, at equilibrium it would charge a price for its good which makes all the combinations but the best one unsalable, so that the other market will also be a monopoly. This monopoly outcome in the adjacent market will prevail even if the distribution of consumer tastes satisfies the condition for covered-duopoly market in a single-market setting. Namely, if we assume that the smallest consumer taste is less than four times and more than twice smaller than the largest consumer taste. Furthermore, with this condition fulfilled, both markets will not be covered at equilibrium which is to say that at least the lowest-taste consumer will prefer the free outside (non-market) option to the only available top-quality combination.

In a setup with two entrants of each type, the producers of the type of good with the more differentiated qualities will have more market power. Again, as in the single-entrant market setting, these producers might find it optimal under certain conditions to charge prices at which the combinations formed by high-quality good of the one type and low-quality good of the other type will not have positive market shares. Such an outcome where these mixed-quality combinations are effectively driven out from the market could occur at equilibrium only if the goods of the two types are not perfect substitutes in qualities. Also, the differentiation between the high and low qualities of each type should be large, namely the quality of the second-best combination must exceed the median between the best combination and the worst combination. The markets, however, could be covered by the combination formed by the low-quality goods of both types.

Further, we explore the conditions for having exactly two entrants of a type at equilibrium given free market entry for the both types of firms. We show that a stricter condition on the consumer taste distribution must be imposed than the one derived for the single-market case. In particular, for the efficient foreclosure of a third entrant, the upper bound on its quality choice needs to be exceeded at least twice by the upper bound on the choice of the firm supplying the best-quality good of the same type.

The market exclusion of the mixed-quality combinations as a result of the pricing policy of the producers of the complementary goods that form them provides a new explanation of the self-selection bias in consumption observed in some industries where there are no tying arrangements (e.g. business-trip transportation and accommodation, real-estate and furniture, PC hardware and software). The established market equilibrium also presents an alternative benchmark case to that of the classical monopolistic competition outcome commonly used in the existing literature on product tying and bundling. Thus, it gives a new perspective for further research on this issue for adjacent vertically differentiated markets where product tying arrangement is imposed by a firm with monopoly power in one of the markets. As shown in the present paper, tying is not needed by the monopolist to deter the entry of a potential competitor in the other market.

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Mathematical Proofs

A Proof of Proposition 1

The proof of the sufficient condition for having at most two entrants of type B at equilibrium follows the procedure suggested by Shaked and Sutton (1982). However, here we apply it to the model setting with two types of complementary goods, single entrant of type A and consumers identified by taste but not by income.

Suppose that all n entrants of type B in the market have positive market shares. The respective profit-maximization problems that they solve at the pricing stage could be derived from the expressions in (14) by substituting sequentially for the marginal tastes from the expressions in (6), (7) and the demands in (8), given the ranking in table 1 as follows:

$$
\begin{aligned}
&\max_{p_1^B} \Pi_1^B = p_1^B D_1^B = p_1^B \left(\bar{\theta} - \theta_{11/12}\right) = p_1^B \left(\bar{\theta} - \frac{p_1^B - p_2^B}{\chi_{11} - \chi_{12}}\right) \\
&\max_{p_2^B} \Pi_2^B = p_2^B D_2^B = p_2^B \left(\theta_{11/12} - \theta_{12/13}\right) = p_2^B \left(\frac{p_1^B - p_2^B}{\chi_{11} - \chi_{12}} - \frac{p_2^B - p_3^B}{\chi_{12} - \chi_{13}}\right) \\
&\quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\max_{p_n^B} \Pi_n^B = p_n^B D_n^B = \begin{cases} p_n^B \left(\theta_{1n-1/1n} - \underline{\theta}\right), & \text{if } \theta_{1n/0} \leq \underline{\theta} \\ p_n^B \left(\theta_{1n-1/1n} - \theta_{1n/0}\right), & \text{if } \theta_{1n/0} > \underline{\theta} \end{cases} = \\
&\qquad = \begin{cases} p_n^B \left(\frac{p_{n-1}^B - p_n^B}{\chi_{1n-1} - \chi_{1n}} - \underline{\theta}\right), & \text{if } p_n^B \leq \chi_{1n}\underline{\theta} - p_1^A \\ p_n^B \left(\frac{p_{n-1}^B - p_n^B}{\chi_{1n-1} - \chi_{1n}} - \frac{p_1^A + p_n^B}{\chi_{1n}}\right), & \text{if } p_n^B > \chi_{1n}\underline{\theta} - p_1^A \end{cases}
\end{aligned}
\tag{A1}
$$

The corresponding first-order optimality conditions could be represented by the following system of equations:

$$
\begin{aligned}
&\bar{\theta} - 2\theta_{11/12} - \frac{p_2^B}{\chi_{11} - \chi_{11}} = 0 \\
&\theta_{11/12} - 2\theta_{12/13} - \frac{p_2^B}{\chi_{11} - \chi_{12}} - \frac{p_3^B}{\chi_{12} - \chi_{13}} = 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\begin{cases} \theta_{1n-1/1n} & -\underline{\theta} - \frac{p_n^B}{\chi_{1n-1} - \chi_{1n}} = 0, \qquad \text{if } p_n^B \leq \chi_{1n}\underline{\theta} - p_1^A \\ \theta_{1n-1/1n} & -\theta_{1n/0} - \frac{p_n^B}{\chi_{1n-1} - \chi_{1n}} - \frac{p_n^B}{\chi_{1n}} = 0, \ \text{if } p_n^B > \chi_{1n}\underline{\theta} - p_1^A \end{cases}
\end{aligned}
\tag{A2}
$$

The system of first-order optimality conditions in (A2) implies the following inequality relations of the marginal taste variables at equilibrium:

$$
\bar{\theta} > 2\theta_{11/12} > 4\theta_{12/13} > ... > 2^{k-1}\theta_{1k-1/1k} > ... > \begin{cases} 2^{n-1}\underline{\theta}, & \text{if } p_n^B \leq \chi_{1n}\underline{\theta} - p_1^A \\ 2^{n-1}\theta_{1n/0}, & \text{if } p_n^B > \chi_{1n}\underline{\theta} - p_1^A \end{cases}
\tag{A3}
$$

Hence, when condition (15) holds, we should have the following inequality fulfilled:

$$
\theta_{12/13} < \underline{\theta}
\tag{A4}
$$

which implies zero market share for the entrants with quality rank larger than two. Therefore, when condition (15) is satisfied at most two entrants of type B will enter the market at equilibrium. Q.E.D.

B Proof of Proposition 2

Suppose one entrant of type A and n entrants of type B all with positive market shares. The profit-maximization problem that the single entrant of type A solves at the pricing stage could be derived from (13). As in appendix A above, we derive it by substituting for the demand from (8) and for the marginal tastes from the expressions in (6) and (7) according to the ranking in table 1 as follows:

$$\max_{p_1^A} \Pi_1^A = p_1^A D_1^A = p_1^A \left(\bar{\theta} - \theta_{1n/0}\right) = p_1^B \left(\bar{\theta} - \frac{p_1^A + p_n^B}{\chi_{1n}}\right) \tag{B1}$$

We first assume non-covered market at equilibrium to show that when such an equilibrium exists there cannot be more than a single entrant with positive market share.

The corresponding first-order optimality condition looks as follows:

$$\bar{\theta} - 2\theta_{1n/0} + \frac{p_n^B}{\chi_{1n}} = 0 \tag{B2}$$

The equation in (B2) implies the following inequality that must hold for $\theta_{1n/0}$ at equilibrium:

$$\theta_{1n/0} \geq \frac{\bar{\theta}}{2} \tag{B3}$$

which implies non-covered market outcome only if (17) holds.

Furthermore, combining (B3) with (A3) yields the following result for the marginal tastes at equilibrium:

$$\theta_{1n/0} > \theta_{11/12} > 2\theta_{12/13} > ... > 2^{k-2}\theta_{1k-1/1k} > ... > 2^{n-1}\theta_{1n-1/1n} \tag{B4}$$

which precludes the existence of a non-covered market equilibrium for $n > 1$. Exactly one entrant of type B could have positive market share if a non-covered equilibrium exists.

The condition in (17), however, is only necessary but not sufficient for the existence of a non-covered equilibrium. The sufficient condition is the payoff of the entrant of type A to be higher in case of non-covered market than in the case of covered market. So, now we derive these payoffs explicitly.

Since the existence of a non-covered market equilibrium implies a single entrant in market B, there is only one combination available, A_1B_1. Accordingly, the entrant in market B faces the same profit-maximization problem as the entrant in market A:

$$\max_{p_1^A} \Pi_1^A = p_1^A \left(\bar{\theta} - \frac{p_1^A + p_1^B}{\chi_{11}}\right) \max_{p_1^B} \Pi_1^B = p_1^B \left(\bar{\theta} - \frac{p_1^A + p_1^B}{\chi_{11}}\right) \tag{B5}$$

Therefore, the solution for their prices is also symmetric:

$$p_1^{A*} = p_1^{B*} = \frac{\bar{\theta}\chi_{11}}{3} \tag{B6}$$

By substituting for the prices from (B6) in the expression for the higher-quality marginal taste variable of combination A_1B_1 we show it to be given by a fixed two-thirds share of the highest consumer taste $\overline{\theta}$:

$$\theta_{11/0}^* = \frac{p_1^{A*} + p_1^{B*}}{\chi_{11}} = \frac{2}{3}\overline{\theta} \tag{B7}$$

Hence, the necessary condition in (17) for the non-covered assumption to hold at equilibrium could be strengthened as follows:

$$\underline{\theta} < \frac{2}{3}\overline{\theta} \tag{B8}$$

Finally, we derive the payoff of the entrant of type A by substituting for the optimal prices from (B6) in the expression for Π_1^A in (B5):

$$\Pi_1^{A*} = \frac{\overline{\theta}^2}{9}\chi_{11} \tag{B9}$$

The existence of a covered-market equilibrium where all the n entrants of type B have positive market shares implies the following inequality to hold:

$$\theta_{1n/0} = \frac{p_1^{A**} + p_n^{B**}}{\chi 1n} \leq \underline{\theta} \tag{B10}$$

where p_1^{A**} stands for the optimal price of the good of the entrant of type A given the market is covered by combination A_1B_n while we denote by p_n^{B**} the optimal price of the good of the lowest-ranked actual entrant in market B.

We could re-write the inequality in (B10) by leaving only p_1^{A**} on the left-hand side:

$$p_1^{A**} \leq \underline{\theta}\chi 1n - p_n^{B**} \tag{B11}$$

After setting the price of the lowest-ranked good of type B to be equal to the zero production cost of its producer, the right-hand side of (B11) gives us the value of the maximal price the entrant of type A could charge its good while still preserving the coveredness of the market:

$$\overline{p}_1^{A**} = \underline{\theta}\chi_{1n} \tag{B12}$$

Since at a covered-market equilibrium, all consumers buy the good of the entrant of type A, its maximal revenue is given by the following expression:

$$\Pi_1^{A**} = \frac{\overline{\theta}^2}{9}\chi_{11} \tag{B13}$$

Comparing the expressions for the profit of the entrant of type A in (B9) and (B13) allows us to derive the condition for the former to exceed the latter:

$$\chi_{1n} < \frac{\overline{\theta}^2}{9\left(\overline{\theta} - \underline{\theta}\right)\underline{\theta}}\chi_{11} \tag{B14}$$

Note that the fraction in front of χ_{11} on the right-hand side of the inequality in (B14) is strictly decreasing in $\underline{\theta}$ for $\underline{\theta}$ satisfying the condition in (17):

$$\frac{\partial\left(\frac{\overline{\theta}^2}{9(\overline{\theta}-\underline{\theta})\underline{\theta}}\right)}{\partial\underline{\theta}} = -\frac{\overline{\theta}^2(\overline{\theta} - 2\underline{\theta})}{9(\overline{\theta} - \underline{\theta})^2\underline{\theta}^2} < 0, \text{ for } \underline{\theta} < \frac{\overline{\theta}}{2} \tag{B15}$$

Hence, we could derive the lowest value this fraction could have by substituting for $\underline{\theta} = \frac{\overline{\theta}}{2}$ in (B14). Then, the condition for the profit of the entrant of type A to be larger at a non-covered equilibrium in (B14) could be re-written as follows:

$$\chi_{1n} < \frac{4}{9}\chi_{11} \tag{B16}$$

Now for proving the result in proposition 2, it only remains to check for which of the three cases of substitutability between the qualities of the two types of goods the condition in (16) is sufficient for the inequality in (B16) to be satisfied.

Beforehand, however, note that since $\chi_{1n} < \chi_{11}$ by definition, the inequality in (B14) would always hold as long as the numerator of the fraction in front of χ_{11} exceeds its denominator. This implies that we could have covered-market equilibrium only if the spread of consumer tastes is restricted from below as follows:

$$\overline{\theta}^2 \leq 9\left(\overline{\theta} - \underline{\theta}\right)\underline{\theta} \quad \Rightarrow \underline{\theta} \quad \leq \frac{\overline{\theta}}{7} \tag{B17}$$

Then, the result in (A3) implies that when (B16) holds at most three entrants could have positive market shares. Thus, since the inequality in (B16) is a necessary condition for having a covered-market equilibrium, we cannot have more than three actual entrants at such an equilibrium.

Below, we translate the sufficient condition in (B16) for having non-covered monopoly equilibrium outcome in market B into a particular condition on the maximal-quality constraints of the potential entrants for each of the three cases of substitutability between the qualities of the two types of goods.

First, if the two types of goods are perfect substitutes in qualities ($\sigma \to \infty$), the inequality in (B16) takes the following form:

$$9f\left(A_1\right) + 9g\left(B_2\right) < 4f\left(A_1\right) + 4g\left(B_1\right) \quad \Rightarrow \quad f\left(A_1\right) < \frac{4g\left(B_1\right) - 9g\left(B_2\right)}{5} \tag{B18}$$

which can never be satisfied as long as the condition $\left(f\left(A_1\right) > g\left(B_1\right)\right)$ for having the ranking of the combinations in table 1 holds.

Second, if the two types of goods are imperfect substitutes in qualities ($\sigma = 1$), the inequality in (B16) takes the following form:

$$9f\left(A_1\right)g\left(B_2\right) < 4f\left(A_1\right)g\left(B_1\right) \quad \Rightarrow \quad g\left(B_1\right) > \frac{9}{4}g\left(B_2\right) \tag{B19}$$

Third, if the two types of goods are perfect complements in qualities ($\sigma \to 0$), the inequality in (B16) takes the following form:

$$9g\left(B_2\right) < 4g\left(B_1\right) \quad \Rightarrow \quad g\left(B_1\right) > \frac{9}{4}g\left(B_2\right) \tag{B20}$$

The inequality conditions in (B19) and (B20) are trivially satisfied if the condition in (16) holds. Hence, condition (16) is sufficient for having a monopoly non-covered market outcome at equilibrium provided that the spread of consumer tastes satisfies the necessary condition in (B8). When the latter condition does not hold $\left(\text{i.e. if } \underline{\theta} \geq \frac{2}{3}\overline{\theta} > \frac{1}{2}\overline{\theta}\right)$, however, the result in (A3) implies that the market will be covered by the best-ranked combination A_1B_1 at equilibrium. Therefore, condition (16) is sufficient for having a monopoly market outcome independent on whether the necessary condition in (16) is satisfied. That is, its weaker statement in (17) just ensures that the monopoly outcome will occur at a non-covered market equilibrium. Q.E.D.

C Proof of Proposition 3

In this section of the appendix, we derive the conditions for the following inequality to hold:

$$\chi_{12} > \chi_{21} \tag{C1}$$

i.e. A_1B_2 to be preferred to A_2B_1, at all the three special cases of the quality aggregation function.

Case 1: $f(A_i)$ and $g(B_j)$ - perfect substitutes ($\rho \to 1$)

First, let's suppose that the qualities of goods of type A and the goods of type B are perfect substitutes. That is, the quality of any combination $\mathrm{A}_i\mathrm{B}_j$ is given as a linear sum of the qualities of the goods of types A and B that form it: $\chi_{ij} = f(A_i) + g(B_j)$

Substituting for the combination qualities in (C1) yields the following result:

$$f(A_1) + g(B_2) > f(A_2) + g(B_1) \tag{C2}$$

which after rearrangement takes the form below:

$$f(A_1) - f(A_2) > g(B_1) - g(B_2) \tag{C3}$$

i.e. the difference between the qualities of the goods of type A should exceed the difference between the qualities of the goods of type B.

Case 2: $f(A_i)$ and $g(B_j)$ – non-perfect substitutes ($\rho \to 0$)

Second, let's assume that the goods of type A and the goods of type B are non-perfect substitutes. That is, the qualities of the combinations that any pair of goods of types A and B form are given by the product of their qualities (particular Cobb-Douglas functional form): $\chi_{ij} = f(A_i)\, g(B_j)$

Substituting for the combination qualities in (C1) yields the following result:

$$f(A_1)\, g(B_2) > f(A_2)\, g(B_1) \tag{C4}$$

which after rearrangement takes the form below:

$$\frac{f(A_1)}{f(A_2)} > \frac{g(B_1)}{g(B_2)} \tag{C5}$$

i.e. the ratio between the high quality and the low quality of type A should exceed the respective ratio of the qualities of the goods of type B.

Case 3: $f(A_i)$ and $g(B_j)$ – perfect complements ($\rho \to -\infty$)

Third, let's assume that the goods of type A and the goods of type B are perfect complements. That is, the qualities of the combinations that any pair of goods of types A and B form are given by the smaller of their qualities (particular Leontief functional form): $\chi_{ij} = \min[f(A_i), g(B_j)]$

Substituting for the combination qualities in (C1) yields the following result:

$$\min[f(A_1), g(B_2)] > \min[f(A_2), g(B_1)] \tag{C6}$$

which is equivalent to the following sufficient condition:

$$g(B_2) > f(A_2) \tag{C7}$$

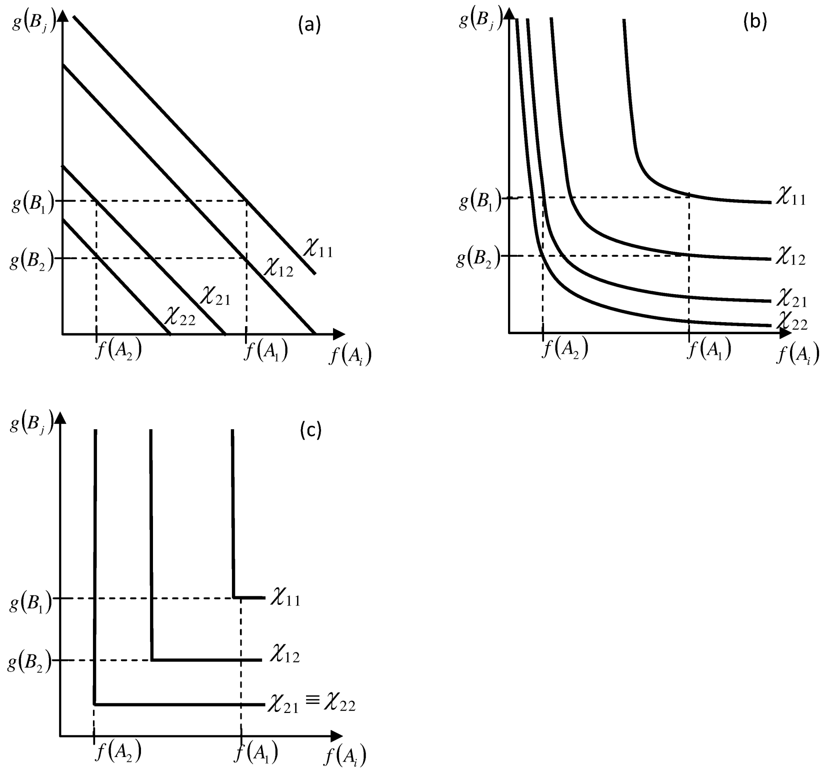
i.e. the low quality of type B should exceed the low quality of type A.

The quality ranking in (18) is sufficient for all the three conditions (C3), (C5) and (C7) to be fulfilled. Hence, for having (C1) satisfied, it is sufficient the quality ranking in (18) to hold. Q.E.D.

The corresponding combination rankings are represented graphically by iso-quality curve mapping for each of the three cases on figure 4 below.

For any given elasticity of substitution between the two types of goods, each iso-quality

Figure 4: Iso-quality curve maps of the combination quality ranking corresponding to the good quality ranking in (18) for: (a) perfect substitutes, (b) non-perfect substitutes, (c) perfect complements.



curve represents the set of pair values $(f(A_i), g(B_j))$ which characterize combinations with equal quality (rank). The more up and to the right an iso-quality curve is located, the higher is the quality (rank) of the combinations formed by the pair values that belong to the curve. As long as the values of the two qualities of type B lay in the interval $(f(A_2), f(A_1))$, qualities of type A are determinative for the ranks of the combinations they form. That is, A_1B_2 is better than A_2B_1 ($\chi_{12} > \chi_{21}$), at any of the three considered cases of constant elasticity of substitution between the two types of goods.

Note that if we swap the values between the axes of any of the graphs on figure 4, the two types of goods will trade their roles. The qualities of type B will become determinative for the ranks of the combinations they form and A_2B_1 will be better than A_1B_2 ($\chi_{21} > \chi_{12}$).

D Proof of Proposition 4

Here, we derive the solutions for the subgame equilibria at the quality-choice and at the pricing stages of the game, respectively as established in proposition 4. The equilibrium solutions are derived by backward induction. Therefore, we start with the solution for the pricing stage.

The number of entrants is exogenously set to two of each type. For now we assume that the qualities of goods are related according to the condition in (18). Latter we will show how the results change if the condition in (19) holds instead. The condition in (18) implies that the combinations formed by the four entrants are ranked by quality according to ranking 1 in table 2. We are interested in establishing an equilibrium at which only the best-quality and worst-quality combinations, A_1B_1 and A_2B_2, have positive market shares at equilibrium. At the equilibrium established in proposition 4 the market is covered, i.e. $\theta_{22/0} < \underline{\theta}$. All the assumptions together imply the following expressions for the profit-maximization problems with respect to prices:

$$\begin{aligned}
\max_{p_1^A} \Pi_1^A &= p_1^A D_1^A = p_1^A \left(\bar{\theta} - \theta_{11/22}\right) = p_1^A \left(\bar{\theta} - \frac{p_1^A + p_1^B - p_2^A - p_2^B}{\chi_{11} - \chi_{22}}\right) \\
\max_{p_1^B} \Pi_1^B &= p_1^B D_1^B = p_1^B \left(\bar{\theta} - \theta_{11/22}\right) = p_1^B \left(\bar{\theta} - \frac{p_1^A + p_1^B - p_2^A - p_2^B}{\chi_{11} - \chi_{22}}\right) \\
\max_{p_2^A} \Pi_2^A &= p_2^A D_2^A = p_2^A \left(\theta_{11/22} - \underline{\theta}\right) = p_2^A \left(\frac{p_1^A + p_1^B - p_2^A - p_2^B}{\chi_{11} - \chi_{22}} - \underline{\theta}\right) \\
\max_{p_2^B} \Pi_2^B &= p_2^B D_2^B = p_2^B \left(\theta_{11/22} - \underline{\theta}\right) = p_2^B \left(\frac{p_1^A + p_1^B - p_2^A - p_2^B}{\chi_{11} - \chi_{22}} - \underline{\theta}\right)
\end{aligned} \tag{D1}$$

The two high-quality producers of type A and B face identical problems. The same holds true also for the low-quality producers of type A and B. Therefore, the solutions for the optimal prices are symmetric:

$$\begin{aligned}
p_1^{A*} = p_1^{B*} &= \frac{3\bar{\theta} - 2\underline{\theta}}{5} \left(\chi_{11} - \chi_{22}\right) \\
p_2^{A*} = p_2^{B*} &= \frac{2\bar{\theta} - 3\underline{\theta}}{5} \left(\chi_{11} - \chi_{22}\right)
\end{aligned} \tag{D2}$$

Substituting for the prices back in the expressions for the profits yields the following expressions for the optimal profits:

$$\begin{aligned}
\Pi_1^{A*} = \Pi_1^{B*} &= \frac{\left(3\bar{\theta} - 2\underline{\theta}\right)^2}{25} \left(\chi_{11} - \chi_{22}\right) \\
\Pi_2^{A*} = \Pi_2^{B*} &= \frac{\left(2\bar{\theta} - 3\underline{\theta}\right)^2}{25} \left(\chi_{11} - \chi_{22}\right)
\end{aligned} \tag{D3}$$

Note that the condition in (17) is stricter than the necessary condition for A_2B_2 to have positive market share:

$$D_{22} = \theta_{11/22} - \underline{\theta} = \frac{2\bar{\theta} - 3\underline{\theta}}{5} > 0 \; for \; \underline{\theta} < \frac{2\bar{\theta}}{3} \tag{D4}$$

Both the expressions in (D3) are increasing in χ_{11} and decreasing in χ_{22}. Also, for any ρ both χ_{11} and χ_{22} are strictly non-decreasing in the qualities of the goods that form them. Hence, the equilibrium solutions for the optimal qualities are given by the upper-bound (maximal quality) constraints for the high-quality goods and by the lower-bound constraints for the low-quality goods:

$$\begin{aligned}
f^*\left(A_1\right) = \bar{F}_{m_1}; f^*\left(A_2\right) = \underline{F} \\
g^*\left(B_1\right) = \bar{G}_{n_1}; g^*\left(B_2\right) = \underline{G}
\end{aligned} \tag{D5}$$

Without loss of generality, we could assume that the two entrants of each type are exactly the potential entrants with highest maximal quality constraints. Then, we could mark the optimal quality choices above by the indices of the initial sequence of maximal quality constraints assigned at the entry stage:

$$\begin{aligned} f^*(A_1) &= \bar{F}_1; f^*(A_2) = \underline{F} \\ g^*(B_1) &= \bar{G}_1; g^*(B_2) = \underline{G} \end{aligned} \tag{D6}$$

Accordingly, by substituting for the optimal quality choices from (D6) in condition (18), it takes the following form:

$$\bar{F}_1 > \bar{G}_1 > \underline{G} > \underline{F} \tag{D7}$$

The results so far are based on the assumption that only the best-quality and worst-quality combinations, A_1B_1 and A_2B_2, have positive market shares at equilibrium. To hold at equilibrium they should imply that the mixed-quality combinations, A_1B_2 and A_2B_1, have no positive market shares at the prices expressed in (D2).

Note that the symmetric prices for the high-quality and low-quality goods of both types imply that the two middle quality cost the same:

$$p_1^{A*} + p_2^{B*} = p_2^{A*} + p_1^{B*} \tag{D8}$$

However, when (D7) holds, A_1B_2 is strictly preferred to A_2B_1. Hence, nobody would buy A_2B_1 at the optimal prices of the goods that form it.

Then, for the sales of A_1B_2 to be foreclosed, the following condition must hold:

$$D_{12} = \theta_{11/12} - \theta_{12/22} \leq 0 \tag{D9}$$

Substituting for the optimal prices from (D2) in the expressions for $\theta_{11/12}$ and $\theta_{12/22}$ yields the following explicit expressions in terms of the combination qualities:

$$\begin{aligned} \theta_{11/12}^* &= \frac{p_1^{B*} - p_2^{B*}}{\chi_{11} - \chi_{12}} = \frac{\left(\bar{\theta} + \underline{\theta}\right)\left(\chi_{11} - \chi_{22}\right)}{5\left(\chi_{11} - \chi_{12}\right)} \\ \theta_{12/22}^* &= \frac{p_1^{A*} - p_2^{A*}}{\chi_{12} - \chi_{22}} = \frac{\left(\bar{\theta} + \underline{\theta}\right)\left(\chi_{11} - \chi_{22}\right)}{5\left(\chi_{12} - \chi_{22}\right)} \end{aligned} \tag{D10}$$

For (D9) to hold, the combination qualities must satisfy the following inequality:

$$\chi_{12} \leq \frac{\chi_{11} + \chi_{22}}{2} \tag{D11}$$

Next, we need to check how this condition on the combination qualities translates into corresponding condition on the good qualities for different values of the constant elasticity of substitution σ. We consider again the three standard cases.

<u>Case 1:</u> $f(A_i)$ and $g(B_j)$ - perfect substitutes $(\rho \to 1)$

When $\sigma \to \infty$ i.e. $\rho \to 1$ and the quality accumulation function is linear, the inequality in (D12) takes the following form:

$$f(A_1) + g(B_2) \leq \frac{f(A_1) + g(B_1) + f(A_2) + g(B_2)}{2}, \ \sigma \to \infty \tag{D12}$$

which holds only when the inequality below is satisfied:

$$f(A_1) - f(A_2) \leq g(B_1) - g(B_2) \tag{D13}$$

The condition in (D13) contradicts not only (18) but also (C3). Therefore, the sales

of combination A_1B_2 cannot be efficiently foreclosed if $\sigma \to \infty$ and the two types of good are perfect substitutes.
Case 2: $f(A_i)$ and $g(B_j)$ – non-perfect substitutes ($\rho \to 0$)

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (D13) takes the following form:

$$f(A_1)\,g(B_2) \leq \frac{f(A_1)\,g(B_1) + f(A_2)\,g(B_2)}{2}, \ \sigma = 1 \tag{D14}$$

which is satisfied only when the following inequality holds:

$$g(B_1) \geq \left(2 - \frac{f(A_2)}{f(A_1)}\right) g(B_2) \tag{D15}$$

Case 3: $f(A_i)$ and $g(B_j)$ – perfect complements ($\rho \to -\infty$)

When $\sigma = 0$ i.e. $\rho \to -\infty$ and the quality accumulation function gives the combination quality as equal to the smaller of the qualities of the goods that form it, the inequality in (D11) takes the following form:

$$\min\left[f(A_1), g(B_2)\right] \leq \frac{\min\left[f(A_1), g(B_1)\right] + \min\left[f(A_2), g(B_2)\right]}{2}, \ \sigma = 0 \tag{D16}$$

which is consistent with the condition in (18) only if the following inequality holds:

$$g(B_1) \geq 2g(B_2) - f(A_2) \tag{D17}$$

Substituting for the optimal quality choices from (D6) in the conditions in (D15) and (D17) yields the following inequality constraints that should hold for the bounds of the quality choices:

$$\bar{G}_1 \geq \left(2 - \frac{\underline{F}}{\bar{F}}\right) \underline{G} \tag{D18}$$

$$\bar{G}_1 \geq 2\underline{G} - \underline{F} \tag{D19}$$

Finally, we need to check the validity of the assumption for covered market at equilibrium. The condition for covered market is given by the inequality:

$$\theta_{22/0} = \frac{p_2^{A*} + p_2^{B*}}{\chi_{22}} \leq \underline{\theta} \tag{D20}$$

which after substituting for the optimal prices from (D1) and (D2) takes the following form in terms of the combination qualities:

$$\frac{4(\chi_{11} - \chi_{22})\,\bar{\theta}}{6\chi_{11} - \chi_{22}} \leq \underline{\theta} \tag{D21}$$

For the restriction on $\underline{\theta}$ in (15) to be stricter than the constraint in (D21), the following condition must hold:

$$\chi_{11} \leq \frac{3}{2}\chi_{22} \tag{D22}$$

Below, we translate it into conditions on the qualities of the goods only for the case of $\sigma = 1$ and $\sigma = 0$.

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (D22) takes the following form:

$$f(A_1)\,g(B_1) \leq \frac{3f(A_2)\,g(B_2)}{2},\ \sigma = 1 \tag{D23}$$

which is satisfied only when the following inequality holds:

$$g(B_1) \leq \frac{3}{2}\frac{f(A_2)}{f(A_1)}g(B_2) \tag{D24}$$

When $\sigma = 0$ i.e. $\rho \to -\infty$ and the quality accumulation function gives the combination quality as equal to the smaller of the qualities of the goods that form it, the inequality in (D22) takes the following form:

$$\min\left[f(A_1), g(B_1)\right] \leq \frac{3\min\left[f(A_2), g(B_2)\right]}{2},\ \sigma = 0 \tag{D25}$$

which is consistent with the condition in (18) only when the following inequality holds:

$$g(B_1) \leq \frac{3}{2}f(A_2) \tag{D26}$$

Substituting for the optimal quality choices from (D6) in the conditions in (D24) and (D26) yields the following inequality constraints that should hold for the bounds of the quality choices:

$$\bar{G}_1 \leq \frac{3}{2}\frac{\underline{F}}{\bar{F}_1}\underline{G},\ \sigma = 1 \tag{D27}$$

$$\bar{G}_1 \geq \frac{3}{2}\underline{F},\ \sigma = 0 \tag{D28}$$

Combining the conditions in (D18) and in (D27) gives the interval in which the maximal quality constraint of B_2 should lay in order when $\sigma = 1$ the conditions in (15) and (17) to be sufficient for having a solution with exogenously set two entrants of each type so that the mixed-quality combinations have no positive market shares and the market is covered. The interval is given by the following expression:

$$\left(2 - \frac{\underline{F}}{\bar{F}_1}\right)\underline{G} \leq \bar{G}_1 \leq \frac{3}{2}\frac{\underline{F}}{\bar{F}_1}\underline{G},\ \sigma = 1 \tag{D29}$$

which is feasible only if the differentiation between the goods of type A is limited as required by the following condition:

$$\underline{F} > \frac{4}{5}\bar{F}_1 \tag{D30}$$

The analogous condition for $\sigma = 0$ is given by combining the conditions in (D19) and (D28):

$$2\underline{G} - \underline{F} \leq \bar{G}_1 \leq \frac{3}{2}\underline{F},\ \sigma = 0 \tag{D31}$$

which is feasible only when the lower bounds on the qualities of the two types satisfy the following condition:

$$\underline{F} > \frac{4}{5}\underline{G} \tag{D32}$$

The conditions in (D29) and (D30) are stricter than the conditions in (D31) and (D32).

To see that the inequality relations in (23) is sufficient for (D29) and (D30) to be satisfied, note that when $\bar{F}_1 > \frac{6}{5}\underline{G}$ as ensured by (23), the following inequality must hold:

$$\frac{5}{4} > \frac{3}{2}\frac{\underline{G}}{\bar{F}_1} \tag{D33}$$

Hence, having $\frac{3}{2}\frac{\underline{F}}{\bar{F}_1}\underline{G} > \bar{F}_1$ (ensured by (23) together with (D33)) implies that both the constraint from above in (D29) and the inequality in (D30) hold.

Furthermore, when (D30) is satisfied, the following inequality must also hold:

$$\frac{6}{5} > 2 - \frac{\underline{F}}{\bar{F}_1} \tag{D34}$$

Hence, having $\bar{G}_1 > \frac{6}{5}\underline{G}$ which is ensured by (23) together with (D34) implies also that the constraint from below in (D29) holds.

Since the condition in (23) ensures the validity of the conditions in (D29)–(D32) for having covered market with excluded mixed-quality combinations at both $\sigma = 0$ and $\sigma = 1$, we have all the assumptions for the subgame equilibrium solutions in (D2) and (D6) to be fulfilled. That is, the condition in (23) is sufficient for having the solution established in proposition 4. Q.E.D.

Since, the constraints in (19) imply having ranking 2 which is a mirror image of ranking 1, the result in (24) could be derived directly from (23) by trading the respective bounds of the qualities of type A and B.

E Proof of Proposition 5

The single-good conditions on consumer-taste distribution in (15) and (17) are only sufficient but not necessary condition for the solution established in proposition 4.

Indeed, let's assume the less strict condition in (D4) instead of the one in (17) for having positive market share of A_2B_2. If we skip then the condition in (D22) so that $\underline{\theta}$ is limited by the stricter condition for covered market in (D21) instead of being constrained by the restriction in (15), we will finish with having the condition in (25) at which still two entrants of each type could make the same optimal pricing and quality choices. However, since (D22) is not required to hold, we do not need to restrict from above $\bar{G}_1$ and $\bar{F}_1$ in (D29) and (D30). Hence, the condition in (23) could be replaced by the following less restrictive inequality relations between the bounds of the range of the quality choices[7]:

$$\bar{F}_1 > \bar{G}_1 > 2\underline{G} > 2\underline{F} \tag{E1}$$

Correspondingly, the condition in (24) could be replaced by the following more relaxed inequality relations:

$$\bar{G}_1 > \bar{F}_1 > 2\underline{F} > 2\underline{G} \tag{E2}$$

The pairwise comparison of the conditions in (15), (17) and (23) versus the conditions in (25) and (E1) shows that there is a trade-off between the strictness of the conditions on the consumer taste and the strictness of the conditions on the quality bounds for having an efficient foreclosure of the mixed-quality combinations. To be consistent with the existing literature on single-good markets for vertically differentiated products, in proposition 4

[7]Note that the inequality in (E1) is stricter than the inequality in (D18).

we gave preference to the conditions that restrict more the quality-choice bounds than to the ones that limit the consumer-taste distribution.

In proposition 5, however, we assume the condition in (25) because the inequalities in (23) are too restrictive and do not comply with the requirements for having an efficient foreclosure of the sales of an eventual third entrant in the market. The very requirements are derived below to show that the conditions in (26) or (27) are sufficient to ensure only two entrants of a type at equilibrium without violating the condition in (E1). We consider only the cases of perfect complements and non-perfect substitutes, $\sigma = 0$ and $\sigma = 1$, since the case of perfect substitutes $\sigma \to \infty$ was proven to be inconsistent with the conditions for having the mixed-quality combinations excluded from the market.

Suppose we have a third entrant of type A. Let's denote it by A_3. Combination A_3B_1 is the highest-ranked combination that could be formed with A_3. There are two possibilities. A_3B_1 could be of better quality than A_2B_2 or of worse.

For A_3B_1 to be of better quality than A_2B_2, the following condition must hold:

$$\chi_{31} > \chi_{22} \tag{E3}$$

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (E3) takes the following form:

$$f(A_3)\, g(B_1) > f(A_2)\, g(B_2)\,,\ \sigma = 1 \tag{E4}$$

When $\sigma = 0$ i.e. $\rho \to -\infty$ and the quality accumulation function gives the combination quality as equal to the smaller of the qualities of the goods that form it, the inequality in (E1) takes the following form:

$$\min\left[f(A_3)\,, g(B_1)\right] > \min\left[f(A_2)\,, g(B_2)\right]\,,\ \sigma = 0 \tag{E5}$$

which is inconsistent with the condition in (18) for any $f(A_3) < f(A_2)$. That is, we cannot have A_3B_1 of better quality than A_2B_2 when the qualities of the two goods are perfect complements.

Given that the conditions in (25), (E1) and (E4) are satisfied, the market share of A_3B_1 is given by the following expression:

$$D_{31} = \theta_{11/31} - \theta_{31/22} = \frac{p_1^A - p_3^A}{\chi_{11} - \chi_{31}} - \frac{p_3^A + p_1^B - p_2^A - p_2^B}{\chi_{31} - \chi_{22}} \tag{E6}$$

which after substituting for the optimal prices from (E2) takes the form:

$$D_{31}^* = \frac{\left[(\chi_{11} + 2\chi_{31} - 3\chi_{22})\,\bar{\theta} - (4\chi_{11} - 2\chi_{31} - 2\chi_{22})\,\underline{\theta} - 5p_3^A\right](\chi_{11} - \chi_{22})}{5\,(\chi_{11} - \chi_{31})\,(\chi_{31} - \chi_{22})} \tag{E7}$$

The expression in (E7) is negative as long as the following inequality holds:

$$\underline{\theta} > \frac{(\chi_{11} + 2\chi_{31} - 3\chi_{22})}{4\chi_{11} - 2\,(\chi_{31} + \chi_{22})}\,\bar{\theta} \tag{E8}$$

The condition in (D21) is stricter than the one in (E8) given the inequality below:

$$\chi_{31} < \frac{2\chi_{11}^2 - \chi_{11}\chi_{22} + \chi_{22}^2}{2\,(2\chi_{11} - \chi_{22})} \tag{E9}$$

which is fulfilled whenever χ_{31} satisfies the following stricter inequality:

$$\chi_{31} < \frac{\chi_{11}}{2} \tag{E10}$$

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (E10) takes the following form:

$$f(A_3)\,g(B_1) < \frac{f(A_1)\,g(B_1)}{2},\ \sigma = 1 \tag{E11}$$

which holds for any $f(A_1)$ that satisfies the following inequality:

$$f(A_1) > 2f(A_3) \tag{E12}$$

Combining the conditions in (E4) and (E12) yields the following general condition for efficient foreclosure of A_3B_1 when it is better than A_2B_2:

$$f(A_1) > 2f(A_3) > 2\frac{g(B_2)}{g(B_1)}f(A_2) \tag{E13}$$

Note that if we have a third entrant of type B, the same logic would define the condition for efficient foreclosure of A_1B_3 when it is better than A_2B_2 To derive the corresponding condition we should just trade $f(A_i)$ and $g(B_j)$ for $i = j$:

$$g(B_1) > 2g(B_3) > 2\frac{f(A_2)}{f(A_1)}g(B_2) \tag{E14}$$

Alternatively, when the condition in (E4) does not hold so that A_3B_1 is of worse quality than A_2B_2, for efficient foreclosure of the sales of A_3, it is sufficient to have the following inequality satisfied:

$$\underline{\theta} > \theta_{22/31} \tag{E15}$$

which after substituting for the optimal prices from (E2) takes the form:

$$\underline{\theta} > \frac{(\chi_{11} - \chi_{22})\,\bar{\theta}}{4\chi_{11} + \chi_{22} - 5\chi_{31}} \tag{E16}$$

The condition in (D21) is stricter than the one in (E16) as long as the following inequality holds:

$$\chi_{31} < \frac{2\chi_{11} + \chi_{22}}{4} \tag{E17}$$

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (E17) takes the following form:

$$f(A_3) < \frac{f(A_1)}{2} + \frac{f(A_2)\,g(B_2)}{4g(B_1)},\ \sigma = 1 \tag{E18}$$

When $\sigma = 0$ i.e. $\rho \to -\infty$ and the quality accumulation function gives the combination quality as equal to the smaller of the qualities of the goods that form it, the inequality in

(E15) takes the following form:

$$f(A_3) < \frac{f(A_1)}{2} + \frac{g(B_2)}{4},\ \sigma = 0 \tag{E19}$$

Both inequalities in (E18) and (E19) hold for any $g(B_1)$ that satisfies the following inequality:

$$f(A_1) > 2f(A_3) \tag{E20}$$

Note that if we have a third entrant of type B, the same logic would define the condition for efficient foreclosure of B_3 when A_1B_3 is of worse quality than A_2B_2. To derive the corresponding condition we should just swap $f(A_i)$ with $g(B_j)$ in (E19):

$$g(B_1) > 2g(B_3) \tag{E21}$$

The comparison between the inequality constraints in (E13) and (E20) implies that the condition in (E20) is stricter and therefore sufficient for having the sales of A_3 efficiently foreclosed. Similarly, the condition in (E21) is sufficient for having the sales of B_3 efficiently foreclosed.

Both the conditions in (E20) and in (E21), however, are satisfied as long as the maximal quality constraints satisfy whichever of the following inequality relations:

$$\bar{F}_1 > \bar{G}_1 > 2\bar{G}_3 > 2\bar{F}_3 \tag{E22}$$

$$\bar{G}_1 > \bar{F}_1 > 2\bar{F}_3 > 2\bar{G}_3 \tag{E23}$$

Hence, both the conditions in (26) and in (27) ensure that at most two firms of a type will enter the market at equilibrium. Q.E.D.

The inequality relations in (26) combine the conditions in (E1) and (E22) in the same way as the conditions in (E2) and (E23) are combined in the inequality relations in (27). Therefore, the inequality relations in (26) and (27) also ensure efficient foreclosure of the mixed-quality combinations and covered market at equilibrium. The choice of the relations between the maximal quality constraints $\bar{G}_3$ and $\bar{F}_3$ in (E22) and (E23) is not occasional. Indeed, if there is a third entrant, the optimal quality choice of the second-ranked entrant of its type will be set so that it is not possible to be exceeded by the quality choice of the third entrant. For example, if there is a third entrant of type A, the optimal choice for A_2 will be slightly above the maximal quality constraint of A_3:

$$f^*(A_2) = \bar{F}_3 + \varepsilon, \varepsilon > 0, \varepsilon \to 0 \tag{E24}$$

Similarly, if there is a third entrant of type B, the optimal choice for B_2 will be slightly above the maximal quality constraint of B_3:

$$g^*(B_2) = \bar{G}_3 + \varepsilon, \varepsilon > 0, \varepsilon \to 0 \tag{E25}$$

Therefore, for the condition in (18) to be satisfied in case both A_3 and B_3 enter the market at the first stage, $\bar{G}_3$ should exceed $\bar{F}_3$. Alternatively, $\bar{F}_3$ should exceed $\bar{G}_3$ for the condition in (19) to be satisfied under the same circumstances.

Abstrakt

Klasická literatura vertikální diferenciace předpokládá separátně poptávané komodity nabízené samostatnými firmami. Tento článek opouští zmiňovaný klasický předpoklad a zkoumá cenovou konkurenci na vertikálně diferenciovaném trhu, kde produkt každé firmy není konzumován separátně, ale je konzumován v páru spolu s dalším jedním komplementárním produktem jiného producenta. Ukazujeme, že na trzích s dvěma potenciálními firmami pro každý produkt může existovat rovnováha, v které produktové páry se smíšenou kvalitou, sestávající se z jednoho kvalitního a jednoho nekvalitního produktu, nejsou poptávány. Tento rovnovážný stav existuje, pakliže je produktový pár se smíšenou kvalitou diferenciován od nejkvalitnějšího produktového páru více, než je diferenciován od nejnekvalitnějšího produktového páru, který má pozitivní podíl na trhu. Tato exkluze produktových párů se smíšenou kvalitou tvoří další způsob jak implementovat známý princip maximální diferenciace v prostředí s více trhy a také vysvětluje samo-selekci pozorovanou na trzích s komplementárními produkty.

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