Working Paper Series **621**
(ISSN 1211-3298)

Selective Sampling with Information-Storage Constraints

Philippe Jehiel
Jakub Steiner

CERGE-EI
Prague, July 2018

ISBN 978-80-7343-428-1 (Univerzita Karlova, Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-471-6 (Národohospodářský ústav AV ČR, v. v. i.)

Selective Sampling with Information-Storage Constraints*

Philippe Jehiel
PSE and UCL

Jakub Steiner
University of Edinburgh and CERGE-EI†

May 9, 2018

Abstract

A decision-maker acquires payoff-relevant information until she reaches her storing capacity, at which point she either terminates the decision-making and chooses an action, or discards some information. By conditioning the probability of termination on the information collected, she controls the correlation between the payoff state and her terminal action. We provide an optimality condition for the emerging stochastic choice. The condition highlights the benefits of selective memory applied to the extracted signals. The constrained-optimal choice rule exhibits (i) confirmation bias, (ii) speed-accuracy complementarity, (iii) overweighting of rare events, and (iv) salience effect.

Keywords: bounded rationality, cognitive constraints, information processing, stochastic choice, confirmation bias, speed-accuracy complementarity, probability weighting, salience.

JEL codes: D03, D80, D81, D83, D89, D90.

---

*This paper has been previously circulated under the title: "On Second Thoughts, Selective Memory, and Resulting Behavioral Biases." We thank Mark Dean, Alessandro Pavan, Philip Reny, Larry Samuelson, Colin Stewart, Balazs Szentes, colleagues at the University of Edinburgh and the audiences at Bocconi, Queen Mary, Columbia, Ecole Polytechnique, Zurich and St Andrews universities and workshops and conferences in Erice, Alghero, Faro, Gerzensee, New York, Cambridge, and Vancouver. Ludmila Matysková and Jan Šedek provided excellent research assistance. Deborah Nováková and Laura Straková have helped with English. Jakub Steiner has received financial support from the Czech Science Foundation grant 16-00703S. Philippe Jehiel thanks the ERC grant n° 742816 for funding.

†CERGE-EI, a joint workplace of Charles University and the Economics Institute of the Czech Academy of Sciences, Politickych veznu 7, P.O. Box 882, 111 21 Prague 1, Czech Republic.

1 Introduction

Economic agents often acquire information about the state of the economy before making their decisions. The information is typically modelled as a signal that helps the agent refine the distribution of the state and improve the decision-making. Often, signals come over time and agents can absorb only a small number of them. We capture this information-processing friction by assuming that agents can receive as many signals as they wish but can remember only a finite number of them when making their choices. In the simplest setting we analyze, the agent can only remember one signal. A key strategic variable that we consider is to allow the agent to ignore some signals with positive probability and restart the signal extraction process. An agent in the face of the first signal she observes can either make a choice based on this observation or dispose with the first signal and rerun the very same stochastic signal extraction process. In the face of the second observed signal she can either make a final decision (after the second observation and according to the same strategy that maps instantaneous observations to choices) or rerun the signal extraction again and so on. Specifically, we allow agents to employ an arbitrary stationary decision process that specifies for each possible signal realization a probability with which the agent restarts the process as well as the chosen action in case of termination. We do not impose time constraints and costs in the basic formulation so that the friction comes solely from the limited information-storing capacity of the agent.

We ask ourselves: Should the agent optimally make her choice as soon as she receives the first signal whatever the realization of it is, or could she be better off by rerunning the very same information-acquisition process? Can hesitation—selective repetition of a fixed stochastic decision procedure where the repetition conditions on the procedure's outcome—be welfare-enhancing?

A general insight is that selective rerunning of the primitive decision procedure is typically optimal. To document this most generally, we provide a simple necessary condition satisfied by the optimal rerunning strategy. The result is an interim indifference condition imposed on the agent who has concluded her decision-making with a plan to choose a particular action. Given the recommended action, the agent's posterior expected payoff from implementing this action must be the same as the posterior expected payoff from rerunning the whole decision-making—the whole selective repetitions of the primitive signal extraction—and implementing whichever action the second run of the decision-making will recommend. We refer to this as to the *second-thought-free* condition.

For illustration, consider a binary decision of whether to make an investment of a fixed size. The agent receives payoff 1 if she invests in the good state of the economy, payoff -2 if she invests in the bad state, and receives 0 when she does not invest whatever the state. Both states are a priori equally likely and give rise to a population of good and bad signals, with the share of the good signals at 90% in the good state and 10% in the bad state. The agent draws possibly several signal realizations in sequence but remembers only the last one when making her investment choice. We assume she invests if the last drawn signal was good (and does not invest upon the last bad signal). Observe that the decision rule generated by the immediate termination upon the first signal that

comes in does *not* generate a second-thought-free choice rule: An agent whose first observed signal was good prefers to rerun the decision process, since the new run will either lead to investing again or will give rise to the signal realization that conflicts with the first observation and will lead to not investing. Since, conditional on two conflicting signals, not investing is preferred (because the false-positive investment error is relatively costly), the agent benefits from having second thoughts when the first observed signal is good.

In the sequel, we interpret the probability of terminating the decision process after receiving a particular signal as a search intensity for the signal. A higher probability of termination at a given information set inflates the likelihood that the agent makes the terminal choice at the set. We show that the failure of the second-thought-free condition with uniform search intensity in the above investment decision example indicates that relative to the uniform search, the agent benefits from increasing the search intensity for the bad attribute.

More generally, the second-thought-free condition follows from the first-order condition imposed on the optimal search intensities. In the above example, a marginal increase in the relative search intensity in favor of the bad signal is welfare-enhancing when starting from the immediate termination strategy. Consider a deviation from the immediate termination strategy that consists of repeating the signal draw with a small probability whenever the observed signal is good. This new decision procedure effectively replaces a marginal measure of contingencies in which the agent terminates after observing the good signal with new draws of the signal. The indifference to marginal changes in the search intensities at optimum implies the second-thought-free condition. Given that a typical signal structure combined with the immediate termination strategy would not result in a second-thought-free rule for a generic payoff function, we conclude that some asymmetric termination strategy that differentiates the termination decision according to the last observed signal is generically optimal.

The model provides microfoundations to a range of behavioral stylized facts. The unifying principle of our behavioral insights is the intuition that the agent targets her search towards the type of evidence that would provide her with more informed posteriors under the uniform search. This principle generates *confirmation bias*, since evidence that confirms the agent's prior leads to more informed posterior than does evidence that contradicts the prior. An optimally targeted information search also generates *speed-accuracy complementarity*; that is, accuracy of choice declines with the response time. The effect is generated by the confirmation bias: The agent encountering evidence contradicting her prior is likely to disregard the evidence and to have a second thought. Hence, long response times indicate a surprising state of the world, and the constrained-optimal choice rule commits errors in the surprising state relatively often. *Overweighting of rare events* occurs when the agent's task is to form a probability belief about an event that is known to be rare, such as a flight accident, by observing a random flight outcome. Since observing a flight accident is far more informative about the probability of future accidents than observing an uneventful flight, the agent optimally biases her search towards eventful flights. In the last behavioral application, we show that distinct states of the world are *salient* in the sense that they attract the agent's

attention (i.e., trigger higher termination rates in our framework). The effect arises because an indistinct perception stimulus that can be generated by several similar states is less informative than a distinct perception stimulus likely generated by a specific distinct state. Hence, the optimal information search targets stimuli indicating distinct states.

Related literature

When the decision-maker can choose from feasible error distributions, then choice data partially reveal the decision-maker's objective, since the constrained-optimal choice rule exhibits costly types of error relatively rarely. At various levels of formalization, this error-management idea has appeared repeatedly in biology, psychology and economics; see Johnson et al. (2013) for an interdisciplinary review. A range of economic models make the error-management idea precise by specifying particular sets of feasible error distributions. These models differ greatly in the assumed constraints imposed on decision-making, and hence in the predicted constrained-optimal stochastic choice rules. In what follows we review the variety of the modeling approaches to frictional decision-making.

Sims (2003), Matějka and McKay (2015) and Steiner et al. (2017) constrain the expected reduction of the entropy of the decision-maker's belief within the decision process. The entropy-based models generate constrained-optimal choice rules akin to the logit rules used in structural estimation. The details of the predictions over the stochastic choice are, however, sensitive to the assumed entropy-based constraint. A different information-processing constraint is assumed in sequential-sampling and drift-diffusion models, e.g. Wald (1945), Arrow et al. (1949), Ratcliff (1978), Hébert and Woodford (2016), and Morris and Strack (2017). By choosing the regions of the stopping beliefs, the decision-maker can trade off the accuracy against the speed of her decision procedure. The class of drift-diffusion models restricts the agents to learning procedures with continuously evolving Bayesian beliefs. In contrast to this assumption, Zhong (2017) argues that learning processes with discontinuous belief evolution are optimal under a broad class of information acquisition costs, and Che and Mierendorff (2016) study one such discontinuous learning in which signals arrive in a Poisson process. Yet another modeling approach to limited cognition conceptualizes decision-making and information processing as finite algorithms with adjustable parameters, e.g. Hellmann and Cover (1970), Compte and Postlewaite (2012), and Wilson (2014).

Compared to the above diverse models that fully specify the cognitive friction, our model delivers partial characterization of the constrained-optimal stochastic choice rule without a full specification of the cognitive constraint. Another research program delivering robust predictions about constrained-optimal stochastic behavior without full specification of the cognitive friction is by Caplin et al. (2017) who provide full behavioral characterization of all posterior-separable information cost models. Yet another alternative to our attempt to deliver predictions robust to the detail of the friction is to estimate the information processing cost from the choice data. The proposed methodologies of information cost identification in Caplin and Dean (2015) and Oliveira et al. (2017) make this approach theoretically feasible when rich data, such as choice data over menus or many decision problems, are available.

This paper belongs to a growing economic literature that explains established behavioral phenomena as the constrained-optimal behavior of decision-makers facing information processing frictions. For instance, Robson (2001), Rayo and Becker (2007), Netzer (2009), and Khaw et al. (2017) provide microfoundations for risk attitudes; Gabaix and Laibson (2017) endogenize discounting; and Wilson (2014), Compte and Postlewaite (2012), and Leung (2017) establish constrained-optimal ignorance of weakly informative news. The partitional model of Dow (1991) focuses on the problem of coarsening of rich information available to the decision-maker.[1]

A related model of extreme-events oversampling was developed by psychologists Lieder et al. (2014). Their decision-maker has access to the objective distribution of a payoff state but faces a friction in the computation of the payoff expectations. She forms the expected payoffs for considered actions in a Monte Carlo simulation as a utility average over a finite sample of simulated payoff states. Instead of the uniform sampling from the true belief distribution, she optimally oversamples states in which stakes are high (and corrects for the oversampling by lowering the respective weights), as in the so-called importance-sampling Monte Carlo method, e.g. Glynn and Iglehart (1989). This project and us examine the opposing sides of information processing. While we assume non-representative sampling in the signal collection and allow for frictionless formation of the posterior beliefs, Lieder et al. take available information as given and impose the friction on its interpretation.

Brain imaging technologies have recently advanced to the level that documenting the process of hesitation in real time for non-human subjects is possible. Studies on rats (Redish, 2016) and monkeys (Rich and Wallis, 2016) document that animals in some decision problems deliberate in a stochastic series of action plans, in which their minds move there-and-back between the available options. The researchers are able to link distinct hippocampal activities representing particular parts of a maze, and link in real time those brain patterns to a sequence of deliberation of a rat that has paused and is choosing how to proceed in the maze. Unlike the neural processes represented by the drift-diffusion model of Ratcliff (1978) that assumes a continuous evolution of a mental state, these deliberation processes, like our model, exhibit discrete transitions between the deliberated actions.[2]

2 Model

2.1 General model

An agent faces a decision under uncertainty. She chooses an action $a \in A$ in a process specified below and receives a payoff $u(a, \theta)$ in the fixed payoff state $\theta \in \Theta$ drawn from an interior prior

[1] Somewhat less related is a literature that explores how mistakes in learning can lead to behavioral biases (see in particular the modeling of coarse learning in Jehiel (2005) that can give rise to overoptimism as shown in Jehiel (2017)). By contrast, in our approach, the agent is constrained in the information she can store but is otherwise behaving optimally given the constraint.

[2] The study of Rich and Wallis (2016) suggests that, in contrast with our baseline model but in accord with our extensions in Section 6, information is aggregated across the rounds of deliberation.

distribution $\pi \in \Delta(\Theta)$. The state space Θ and the action set A are finite. The agent chooses a signal structure—or equivalently a Blackwell experiment—p, where p is characterized by a family of conditional signal distributions $p(x \mid \theta)$, $\theta \in \Theta$. The experiment generates a signal x from a finite signal space X. The conditional signal distributions are fully mixed: $p(x \mid \theta) > 0$ for all x, θ. We allow the agent to choose among possibly several such Blackwell experiments and we let $\mathcal{P}$ denote the set of experiments from which she chooses. We impose no restrictions on the set $\mathcal{P}$ of the feasible experiments (other than the full-support of each p).

One interpretation is that each experiment $p \in \mathcal{P}$ is a particular reasoning approach available to the agent. The agent can repeat the selected experiment arbitrarily many times, but she is unable to aggregate the information across the repetitions. Each run of the experiment is a cognition that exhausts the agent's capacity dedicated to the problem being solved. Once the agent hits the constraint at the end of the experiment, she can continue only after she unclogs her capacity by amnesia.

The agent can condition the repetition of the experiment on the last observed signal. She chooses a vector $\beta = (\beta_x)_{x \in X} \in B = [0,1]^{|X|} \setminus \{(0,\ldots,0)\}$ of termination probabilities β_x for each signal realization x; we call β a *termination strategy*. The agent runs the experiment p for the first time, receives signal realization x_1 with probability $p(x_1 \mid \theta)$ and terminates the reasoning with probability β_{x_1}. She restarts her reasoning with the complementary probability $1-\beta_{x_1}$, and receives a signal realization x_2 from a new run of the process p with probability $p(x_2 \mid \theta)$, terminates with probability β_{x_2} or restarts with probability $1-\beta_{x_2}$, and continues to rerun the primitive process p until she terminates after a random number of repetitions of p; see Figure 1. When the agent chooses having distinct β_x for different x, then she implements the familiar idea of selective memory; some facts and observations are easily forgotten whereas others are remembered and they trigger choice. After the agent terminates the reasoning with a terminal signal x, she selects an action $a = \sigma(x)$ according to an *action strategy* $\sigma : X \longrightarrow A$.[3] Let S be the set of all mappings from X to A.

By excluding the termination strategy $(0,\ldots,0)$ we prohibit the agent from avoiding to take the decision $a \in A$. Since $\beta \neq (0,\ldots,0)$ and each feasible experiment p generates all signals with a positive probability in each state, the decision process almost surely eventually terminates. Note that we can accommodate agents who have an outside option by enlarging the action space.

The outcomes of distinct runs of p are conditionally independent. Thus, the probability that the agent terminates after t repetitions of the experiment p resulting in the signal history $\mathbf{x}^t = (x_1,\ldots,x_t)$ is

$$\rho\left(\mathbf{x}^t \mid \theta; p, \beta\right) = \beta_{x_t} p(x_t \mid \theta) \prod_{l=0}^{t-1} \left(1-\beta_{x_l}\right) p\left(x_l \mid \theta\right). \tag{1}$$

We let

$$r(a \mid \theta; p, \beta, \sigma) = \sum_t \sum_{\mathbf{x}^t : \sigma(x_t) = a} \rho\left(\mathbf{x}^t \mid \theta; p, \beta\right) \tag{2}$$

[3] We do not allow for mixed action strategies since the optimum can always be achieved with a pure action strategy.

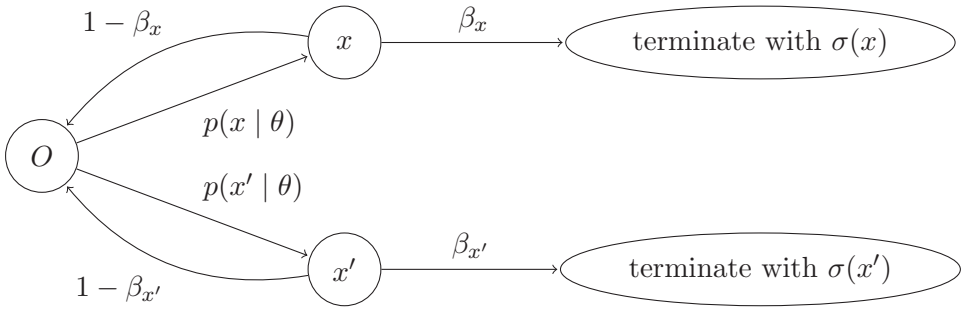

Figure 1: For each (p, β, σ), the decision process is a Markov chain evolving on the agent's states of mind, with transition probabilities that depend on the payoff state θ. The chain begins in the state of mind O and transits to states $x \in X$ with probabilities $p(x \mid \theta)$. The process returns to O with probability $1 - \beta_x$, or terminates with choice of $a = \sigma(x)$ with probability β_x.

denote the probability that the agent who employs the experiment p, the termination strategy β, and action strategy σ terminates with action a. We call $r(p, \beta, \sigma) := (r(a \mid \theta; p, \beta, \sigma))_{a \in A, \theta \in \Theta}$ the *choice rule*. The set of feasible choice rules is $\mathcal{R}(\mathcal{P}) = \{r(p, \beta, \sigma) : p \in \mathcal{P}, \beta \in B, \sigma \in S\}$. Sometimes we abuse notation, omit p, β, σ and write $r(a \mid \theta)$ for the probability of a in state θ under the rule constructed by some p, β, σ.

The *repeated-cognition problem* is to select a feasible choice rule r that maximizes the expected payoff:

$$\max_{r \in \mathcal{R}(\mathcal{P})} \sum_{\theta \in \Theta, a \in A} \pi_\theta r(a \mid \theta) u(a, \theta). \tag{3}$$

The agent solving the repeated-cognition problem knows the prior π, payoff function u, and the set $\mathcal{P}$ of the feasible processes p. The optimization in (3) can be an outcome of selective pressures that favor successful decision procedures via cultural or biological evolution, or via competition of firms differing in their internal procedures.

We focus on the benefit of the repeated cognition in that our baseline model abstracts from the cost of time and is therefore applicable to agents who can repeat the basic cognition process p quickly. The model extends to agents who exponentially discount future payoffs, and thus face non-trivial cost of repeated cognition. We report such an extension in Section 6.2.

2.2 Examples

The first example is the simplest specification of the above baseline model. The reader may focus on this elementary setting during the first reading of the paper.

Example 1 (*elementary setting*). The agent has access only to one statistical experiment: $\mathcal{P} = \{p\}$, where p is an exogenous information structure that specifies conditional signal distributions $p(x \mid \theta)$. In this case, the agent only chooses the termination probabilities β_x, and the action strategy σ that determines the action $a = \sigma(x)$ chosen at each terminal signal x.

Our baseline model, however, accommodates more complex cognitive processes that involve remembering multiple signals, or partial aggregation of information over time. The accommodation of these complex examples within our baseline model is nontrivial and postponed to Section 6.1.

Example 2 (*imperfect information aggregation*). This setting relaxes the agent's inability to aggregate information across the repetitions of her reasoning by endowing her with a finite set of memory states that she can use to represent the signal histories. One interpretation of this example is that the agent keeps track of the number of elapsed rounds, can count up to a finite number, and her count of the rounds is allowed to be stochastic. The setting of this example builds on Hellmann and Cover (1970) and Wilson (2014).

The agent is endowed with one Blackwell experiment $\mu(x \mid \theta)$ with a finite signal space X and, additionally, with a finite set M of the memory states m. After each run of the experiment μ, the agent either terminates or continues with decision-making. If the agent continues, then she transitions from the current memory state to a new memory state and reruns the statistical experiment $\mu(x \mid \theta)$. That is, the agent selects a (generalization of the) termination strategy: $\gamma : M \times X \longrightarrow \Delta(M \cup \{\mathfrak{t}\})$, where $\gamma(m' \mid m, x)$ is the probability that the agent in the memory state m who has observed signal x in the last run of the experiment μ continues with the decision-making and transitions to the memory state m', and $\gamma(\mathfrak{t} \mid m, x)$ is the probability that such an agent terminates. We restrict $\gamma(\mathfrak{t} \mid m, x)$ to be positive for all pairs m, x to ensure that the decision-making almost surely eventually terminates. The terminating agent chooses action $\sigma(m, x)$ that depends both on the current memory state and on the signal acquired in the last run of μ. The agent starts the decision-making in the memory state m_0. A pair γ, σ induces a θ-dependent Markov chain over the memory states that eventually terminates with choice $\sigma(m, x)$, where m is the last memory state and x is the last signal received. Let $p(a \mid \theta; \gamma, \sigma)$ be the probability that the agent terminates with the choice a in state θ, and let $\mathcal{P}_{iia}$ be the set of all stochastic choice rules p that this agent can construct. She selects the choice rule from $\mathcal{P}_{iia}$ that maximizes her ex ante expected payoff.

Although, unlike in our baseline model, the agent of this example is endowed with nontrivial memory, we show in Section 6.1 that the example can be accommodated within our baseline model by a suitable choice of the set $\mathcal{P}$ of the experiments and of the signal space X.

Example 3 (*partial forgetting*). In this example, the agent comprehends up to $N > 1$ signals sequentially drawn from the experiment $\mu(x \mid \theta)$ with finite signal space X. She can discard some of the accumulated data at any interim stage of the decision process. Unlike in the baseline model, she is not restricted to discarding all her information.

Let H be the set of the signal histories h of length $|h| \leq N$. The agent at a history h can (i) terminate her decision-making; (ii) discard some of the information accumulated; or (iii), if $|h| < N$, acquire a new signal. (i) An agent terminating at h chooses action $\sigma(h)$. (ii) An agent who discards some information transitions to a truncation h' of her current history h.[4] (iii) An agent who acquires a new signal transitions to a history hx, where x is the new signal drawn from

[4] A truncation is obtained by deleting one or more last elements in h.

$\mu(x \mid \theta)$.

More precisely, the decision-making is governed by a pair of mappings $\gamma : H \times \Theta \longrightarrow \Delta(H \cup \{\mathfrak{t}\})$ and $\sigma : H \longrightarrow A$, where $\gamma(h' \mid h, \theta)$ stands for the probability that the agent at history h in state θ continues decision-making and transitions to h', and $\gamma(\mathfrak{t} \mid h, \theta)$ is the probability of termination at history h in state θ. The mapping γ is constrained to satisfy 1. $\gamma(h' \mid h, \theta)$ is independent of θ if h' is a truncation of h, 2. $\gamma(\mathfrak{t} \mid h, \theta)$ is independent of θ, 3. $\frac{\gamma(hx|h,\theta)}{\gamma(hx'|h,\theta)} = \frac{\mu(x|\theta)}{\mu(x'|\theta)}$, 4. $\gamma(h' \mid h, \theta) = 0$ unless h' is a truncation of h, or $h' = hx$ for some $x \in X$ and $|hx| \leq N$. Constraints 1 and 2 require the agent to condition the decision to discard information or to terminate only on her current history independently of the state. Constraint 3 allows the agent to expand her information set only by running the experiment $\mu(x \mid \theta)$. Constraint 4 restricts each step of information acquisition to one draw from $\mu(x \mid \theta)$ or to a partial discarding of the accumulated information. Let $p(a \mid \theta; \gamma, \sigma)$ be the probability that the agent who employs (γ, σ) terminates with action a in the state θ. The agent chooses γ and σ to maximize her ex ante expected payoff.

Again, we show in Section 6.1 that this example can be embedded within the baseline model once the set $\mathcal{P}$ of the experiments and the signal space X are suitably specified.

3 Optimal cognition biases

We now derive a necessary optimality condition that the choice rule solving the repeated-cognition problem must satisfy. We argue in Section 3.3 that, generically, the condition requires the agent to engage in selective memory—that is, to ignore some signals more often than others.

3.1 Second-thought-free choice rules

We start with a definition of *second-thought-free* choice rules. If the agent's decision process generates such a rule, then she has no incentive to rerun the process regardless of the action recommendation with which the process terminates. Our main result below states that an optimal rule that solves the repeated-cognition problem is second-thought-free.

Let A and Θ be finite action and state sets with generic elements a and θ. Let r be a generic stochastic choice rule that specifies conditional probabilities $r(a \mid \theta)$ of each action $a \in A$ in each state $\theta \in \Theta$.

Definition 1. *The choice rule r is* second-thought-free *with respect to the utility u if the agent prefers each action recommended by the rule to a new run of the rule r. That is, for each action a chosen with positive probability,*

$$\mathrm{E}_\alpha[u(a, \theta) \mid a_1 = a] \geq \mathrm{E}_\alpha[u(a_2, \theta) \mid a_1 = a], \tag{4}$$

where the expectations are with respect to the random variables θ and a_2, and $\alpha(\theta, a_1, a_2) = \pi_\theta r(a_1 \mid \theta) r(a_2 \mid \theta)$ is the joint probability distribution of the state and two actions consecutively recommended by the rule r.

Consider an agent whose decision process generates choice a with probability $r(a \mid \theta)$ in state θ. The definition requires the agent who terminates her decision process with an action plan a to weakly prefer the action a to forgetting a and choosing whichever action a new run of the decision process will recommend. Although the definition allows for the strict preference against having a second thought, the next lemma shows that (4) is always met with equality: If a choice rule is second-thought-free, then the agent is indifferent between terminating and the second thought. The lemma is a simple consequence of the law of iterated expectations.

Lemma 1. *If a choice rule r is second-thought-free, then the condition* (4) *is met with equality for each action a chosen with positive probability:*

$$\mathrm{E}_\alpha[u(a,\theta) \mid a_1 = a] = \mathrm{E}_\alpha[u(a_2,\theta) \mid a_1 = a]. \tag{5}$$

All proofs are relegated to Appendix. We refer to (5) as the *second-thought-free condition*.

3.2 Optimality condition

Our main result follows. Note that the existence of the solution to the repeated-cognition problem is not guaranteed since we do not impose any restrictions on the set $\mathcal{P}$ of the feasible experiments. We prove existence for a binary setting in Section 4, where we impose more structure on the model.

Proposition 1. *If a choice rule solves the repeated-cognition problem* (3)*, then it is second-thought-free, and satisfies* (5)*.*

To understand the statement, consider the optimal choice rule r^* generated by a process that consists of a random number of repetitions of a primitive cognition p. Once these repetitions of p terminate with a terminal signal x and the agent is about to take an action $a = \sigma(x)$, then, according to the proposition, she must be indifferent between a, and running the process associated with r^* from scratch, where the new run of r^* would involve new repetitions of p.

To prove Proposition 1, we first introduce an *effective experiment* $s(p,\beta)$ and distinguish it from the *primitive experiment* p. While $p(x \mid \theta)$ specifies the probability that one run of the experiment p results in signal x, $s(x \mid \theta; p, \beta)$ is the probability that selective repetitions of p according to the termination strategy β terminate with the signal x. Relative to the primitive probabilities $p(x \mid \theta)$, the effective probabilities $s(x \mid \theta; p, \beta)$ are inflated for those signals x at which the agent terminates with a high probability β_x:

Lemma 2. *The probability that the agent who employs a primitive experiment p and a termination strategy β terminates with signal x in state θ is*

$$s(x \mid \theta; p, \beta) = \frac{\beta_x p(x \mid \theta)}{\sum_{x' \in X} \beta_{x'} p\left(x' \mid \theta\right)}. \tag{6}$$

The simple proof in the appendix exploits the stationarity of the decision process. The lemma implies that $s(p,\beta)$ and hence also $r(p,\beta,\sigma)$ are homogeneous of degree zero with respect to β.

Thus, since we abstract from the delay costs, for any optimal termination strategy β^*, $\alpha\beta^*$ for $\alpha \in (0,1)$ is optimal too, and it generates the same choice rule as β^*.

Lemma 2 can be used to rewrite the agent's objective as an explicit function of the termination strategy: The repeated-cognition problem is equivalent to

$$\max_{p\in\mathcal{P},\beta\in B,\sigma\in S} \sum_{\theta\in\Theta,x\in X} \pi_\theta \frac{\beta_x p(x \mid \theta)}{\sum_{x'\in X} \beta_{x'} p\left(x' \mid \theta\right)} u(\sigma(x),\theta). \tag{7}$$

Proposition 1 follows from the first-order condition with respect to the termination strategy β.

Somewhat counterintuitively, the second-thought-free condition imposes a non-profitability requirement on a non-stationary deviation that is not feasible to the agent. The second-thought-free condition imposed on the rule $r(p^*,\beta^*,\sigma^*)$ induced by the optimal (p^*,β^*,σ^*) considers the agent who has repeated the primitive experiment p^* a stochastic number of times according to the strategy β^*, and is about to terminate for the first time. The condition asks whether the agent can gain by not terminating and by repeating the whole decision process $r(p^*,\beta^*,\sigma^*)$ once and only once. Since our agent is allowed to construct only rules based on stationary termination strategies, this considered deviation is infeasible to the agent. The second-thought-free condition follows from a marginal argument. Consider a marginal decrease of the termination probabilities β_x for all signals x such that $\sigma(x) = a$ to a value $\beta_x(1-\varepsilon)$. This replaces a small measure of contingencies in which the agent terminates the process $r(p,\beta,\sigma)$ with action a with a longer process that consists of two or more repetitions of $r(p,\beta,\sigma)$. The probability of the replacement with k repetitions of $r(p,\beta,\sigma)$ is of the order of ε^k. Thus, the marginal optimality condition only compares the profitability of the termination after the first run of $r(p^*,\beta^*,\sigma^*)$ with its single repetition, and neglects all further repetitions. At optimum, the agent is indifferent between the termination with a and the single repetition of the optimal process.

Comment. Proposition 1 does not make reference to the set $\mathcal{P}$ of the feasible experiments. Rather, the second-thought-free condition is a restriction imposed only on the choice rule r^*, on the prior, and on the utility function. This makes our result relevant to those analysts who do not know the set $\mathcal{P}$ of the feasible primitive cognition processes, but observe the joint distribution of the actions and the payoff states.

3.3 Benefit of cognition biases

For an illustration of how the agent can benefit from a cognition bias that favors termination of the decision-making after encountering a certain types of signal, we review a special case of our elementary setting from Example 1. The agent's task is to announce the realized value of the binary state $\theta \in \{0,1\}$. She is rewarded with payoff $u(a,\theta) = 1$ if her announcement $a \in \{0,1\}$ matches the state θ, and she receives zero otherwise. The state θ is drawn from the uniform prior and the agent has access only to one statistical experiment; $\mathcal{P} = \{p\}$ is a singleton. The available statistical

experiment is asymmetric in that it satisfies

$$.9 = \Pr_p(\theta = 1 \mid x = 1) > \Pr_p(\theta = 0 \mid x = 0) = .6.$$

That is, $x = 1$ (resp. $x = 0$) is informative that the state θ is more likely to be 1 (resp. 0) and signal $x = 1$ is more informative than $x = 0$. Let us fix the action strategy to be the identity function $\sigma_I(x) = x$.[5] With probability .2, the experiment delivers the signal $x = 1$, which provides the agent with the correct advice in 90% of cases, whereas the somewhat less informative signal $x = 0$ that is correct in only 60% of the cases is observed with probability .8.

Consider first an agent who always terminates immediately after the first observed signal: $(\beta_0, \beta_1) = (1, 1) = \mathbf{1}$. For this agent, the choice rule $r(p, \mathbf{1}, \sigma_I)$ and the primitive experiment p coincide, and the agent makes the correct choice with probability $.2 \times .9 + .8 \times .6 = .66$.

The choice rule $r(p, \mathbf{1}, \sigma_I) = p$, however, is not optimal since it fails to be second-thought-free. To verify this, consider the agent who has run the experiment p, receives the muddled signal $x = 0$, and is about to conclude the decision-making with action $a_1 = 0$. Before she implements a_1, let her contemplate having a second thought that involves running the experiment p once again and implementing whichever action $a_2 = x_2$ the new run recommends. If $a_2 = a_1$, then the second thought will have been inconsequential. If $a_2 \neq a_1$, then it is more likely that $\theta = 1$ than $\theta = 0$ (because $x = 1$ is more informative than $x = 0$), and thus the agent benefits from switching her choice from $a_1 = 0$ to $a_2 = 1$. Thus, overall, the agent who is about to terminate the decision process $r(p, \mathbf{1}, \sigma_I)$ with action 0 benefits from having the second thought. By Proposition 1, the process $r(p, \mathbf{1}, \sigma_I)$ is not optimal.

Generically, choice rules are not second-thought-free, and therefore, like in this example, the agent who has access to a generic experiment p and has a generic utility u and a prior π benefits from selective repetitions of the experiment p. In this example, due to the symmetry of the prior and of the utility function, the second-thought-free rule must be symmetric: the optimal termination strategy β^* must satisfy

$$r(1 \mid 1; p, \beta^*, \sigma_I) = \frac{\beta_1^* p(1 \mid 1)}{\beta_0^* p(0 \mid 1) + \beta_1^* p(1 \mid 1)} = \frac{\beta_0^* p(0 \mid 0)}{\beta_0^* p(0 \mid 0) + \beta_1^* p(1 \mid 0)} = r(0 \mid 0; p, \beta^*, \sigma_I). \tag{8}$$

To see why the optimal rule must be symmetric, recall that the second thought affects the payoff only if the two runs of the decision process will result in the conflicting signals $x_1 \neq x_2$. If the rule was not symmetric, then the agent would fail to be indifferent between the two actions, and hence would want to have a second thought after terminating with one of the two actions. Vice versa, when the rule satisfies the symmetry (8), then both states are equally likely in this contingency, and thus the agent is indeed indifferent between the two actions; the symmetric rule is second-thought-free.

The symmetry condition (8) implies that $\beta_0^*/\beta_1^* = 0.15$, and $r(1 \mid 1; p, \beta^*, \sigma_I) = r(0 \mid 0; p, \beta^*, \sigma_I) = 0.79$.[6] The fastest such rule terminates immediately after the agent observes the highly informative

[5] This strategy is shown to be optimal in Section 4.

[6] The computation consists of finding $p(x \mid \theta)$ by reverting the specified posterior probabilities $\Pr_p(\theta \mid x)$, and

signal $x = 1$, but when the agent observes the muddled signal $x = 0$, then she is likely to hesitate: she reruns her cognition p with probability $1 - .15 = .85$. Such a bias in her cognition inflates the likelihood of her decision taking place at the information set $x = 1$ and this modifies the precisions of posteriors under the effective experiment $s(p, \beta^*)$. The probability of the correct choice after terminating at either signal realization becomes 0.79, and thus the overall expected payoff increases to 0.79 compared to only .66 under the unbiased strategy $\beta = \mathbf{1}$.

4 The binary setting

We extend the example studied in subsection 3.3 to cover the general setting with binary action and state spaces. The agent chooses $a \in A = \{0, 1\}$ and receives $u(a, \theta)$, where the payoff state θ is drawn from an interior prior $\pi \in \Delta(\Theta)$, and $\Theta = A$. To avoid a trivial case, we assume that neither action is dominant. Then, without loss of generality, $u(a, \theta) = u_\theta > 0$ if $a = \theta$ and $u(a, \theta) = 0$ otherwise. The set $\mathcal{P}$ of the feasible statistical experiments is finite, and each $p \in \mathcal{P}$ delivers a signal x from a finite signal space X with probability $p(x \mid \theta)$. The agent chooses $p \in \mathcal{P}$, the termination strategy $\beta = (\beta_x)_{x \in X}$ and action strategy $\sigma : X \longrightarrow A$ to maximize the expected payoff as in (3).

The first result states that there exists a solution in which the agent ignores all but two signal realizations of the chosen experiment p. That is, she always repeats the experiment upon encountering all but two signals. Roughly, the result follows because it is advantageous to consider only the most informative signal realizations for each state.[7]

Lemma 3. *There exists a solution in which the termination probability β_x is positive for at most two signal values $x \in X$.*

Based on the lemma, we can, without loss of generality, restrict the signal space X to be binary, and identify it with the action and state spaces, $X = A = \Theta$. Again without loss of generality, we choose signal labels in such a way that each experiment $p \in \mathcal{P}$ satisfies the monotone likelihood ratio property: $p(1 \mid \theta)/p(0 \mid \theta)$ increases in θ. We continue to assume that $p(x \mid \theta) > 0$ for all x and θ.

Recall that σ_I is the identity function, and let the agent employ experiment p and the action strategy σ_I. The next lemma characterizes the set $\mathcal{R}_{p,\sigma_I} = \{r(p, \beta, \sigma_I) : \beta \in B\}$ of the feasible choice rules that such an agent has access to. To characterize this set, we introduce a parameter that we dub *perceptual distance* between states 0 and 1 under the experiment p:

$$d_p = \frac{p(1 \mid 1)p(0 \mid 0)}{p(0 \mid 1)p(1 \mid 0)}.$$

The perceptual distance is a summary statistic of the information structure of the experiment p. The larger it is, the more p reliably discriminates between the two states. The monotone likelihood

solving (8) for β_0^*/β_1^*.

[7] This insight exploits the assumption of perfect patience, since impatient agents would trade off informativeness against delay costs. We conjecture that when exponential discounting is considered, then the result that the agent ignores all but two signal realizations continues to hold for sufficiently patient agents and generic signal structures.

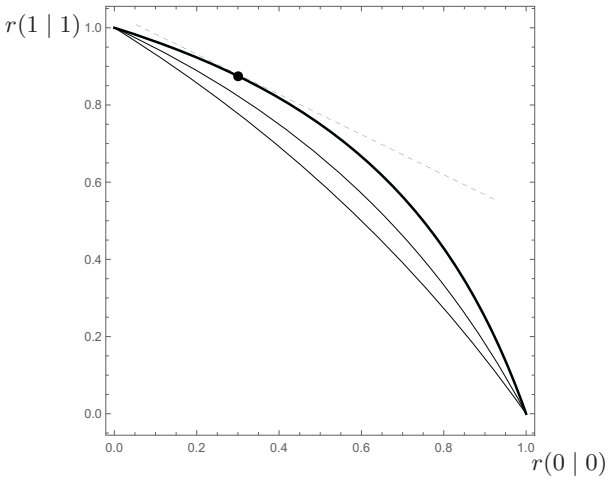

Figure 2: Each point in $[0,1]^2$ on this graph corresponds to a choice rule. The depicted curves are the sets $\mathcal{R}_{p,\sigma_I}$ of the choice rules constructible from experiments p and action strategy σ_I. The thick curve corresponds to the experiment $\overline{p}$ with the maximal perceptual distance $\overline{d}$. Since the objective is linear in the choice rule, the indifference curves are downward sloping lines. The dashed line is the tangential indifference curve. The dot depicts the solution of the repeated-cognition problem.

property of each p implies that $d_p > 1$. The next lemma states that the perceptual distance is preserved under any termination strategy β.

Lemma 4. $\mathcal{R}_{p,\sigma_I} = \{r : r(1 \mid 1)r(0 \mid 0) = d_p r(1 \mid 0)r(0 \mid 1)\}$.

That is, a rule r can be constructed from p if and only if it preserves the perceptual distance: $\frac{r(1|1)r(0|0)}{r(0|1)r(1|0)} = d_p$ (or if it always selects a same action). By controlling the termination strategy β, the agent trades off the likelihoods $r(0 \mid 0; p, \beta, \sigma_I)$ and $r(1 \mid 1; p, \beta, \sigma_I)$ of the correct choice in the states 0 and 1, respectively. See Figure 2. The set $\mathcal{R}_{p,\sigma_I}$ of rules accessible from p is compact.

Thanks to the chosen labeling of the signals, the agent can equate her choice to the observed signal without a loss:

Lemma 5. *For any rule $r(p, \beta, \sigma)$ there exists β' such that the rule $r(p, \beta', \sigma_I)$ achieves at least as high expected payoff as $r(p, \beta, \sigma)$.*

The solution to the repeated-cognition problem in the binary setting exists since the objective is continuous in the choice rule and the agent optimizes on the compact set $\bigcup_{p\in\mathcal{P}} \mathcal{R}_{p,\sigma_I}$ of the rules.

Let $\overline{p}$ be the experiment with the maximal perceptual distance: $\overline{p} \in \arg\max_{p\in\mathcal{P}} d_p$, and let $\overline{d} = \max_{p\in\mathcal{P}} d_p$. In line with the intuition that the agent should go for the most informative experiment, we establish:

Lemma 6. *There exists a solution to the repeated-cognition problem in which the agent employs the experiment $\overline{p}$ with the maximal perceptual distance.*

The last lemma implies that all details of the set $\mathcal{P}$ relevant for the solution are summarized in the one-dimensional statistic $\overline{d}$ that is independent of the payoff function u.[8]

We are now ready to solve the binary setting. The optimal effective choice rule $r^*(a \mid \theta) = r(a \mid \theta; \overline{p}, \beta^*, \sigma_I)$ consists of four unknown probabilities and it is determined by four conditions: the second-thought-free condition (5), the feasibility condition from Lemma 4, and two normalization conditions. Let parameter $R = \frac{\pi_1 u_1}{\pi_0 u_0}$ measure the relative a priori attractiveness of action 1.

Proposition 2. *1. When $R \geq \overline{d}$, then the agent always chooses action 1;*

2. when $R \leq 1/\overline{d}$, then the agent always chooses action 0;

3. when $R \in (1/\overline{d}, \overline{d})$, then the agent chooses both actions with positive probabilities and

$$r^*(1 \mid 1) = \frac{\overline{d}R - \sqrt{\overline{d}R}}{(\overline{d}-1)R},\ r^*(0 \mid 0) = \frac{\overline{d} - \sqrt{\overline{d}R}}{\overline{d}-1}, \tag{9}$$

$$\frac{\beta_1^*}{\beta_0^*} = \frac{\overline{d}R - \sqrt{\overline{d}R}}{\sqrt{\overline{d}R} - R}\frac{\overline{p}(0 \mid 1)}{\overline{p}(1 \mid 1)}. \tag{10}$$

When the ex ante attractiveness of one of the actions is too strong relative to the precision of the best available experiment, then the agent ignores the signals generated by the experiment and always chooses the ex ante attractive action. This arises when the linear indifference line in Figure 2 is steeper than the slope of the set $\mathcal{R}_{\overline{p},\sigma_I}$ at the corners. These slopes at the corners are $-\overline{d}$ and $-1/\overline{d}$, which implies the range $(1/\overline{d}, \overline{d})$ of R for which the optimal rule is interior. In Section 6.1 we show how Proposition 2 extends to cover the agents endowed with more elaborate memory.

The solution of the repeated-cognition problem has natural comparative statics reminiscent of wishful thinking. As the ex ante attractiveness of a state increases, the agent searches more intensively for evidence in support of this state, her relative performance in this state improves, and the relative speed of the decision-making in this state increases. Let $f_\theta = \sum_x \beta_x p(x \mid \theta)$ stand for the decision rate in state θ; it is the per-round probability that the agent in state θ terminates the decision-making. The response time in state θ is geometrically distributed with the expected response time $1/f_\theta$.

Corollary 1. *Let $R \in (1/\overline{d}, \overline{d})$. Then,*

1. relative search intensity β_1^*/β_0^* for signal 1 increases with R;

2. for each state θ, the relative likelihood $\frac{r^*(1|\theta)}{r^*(0|\theta)}$ of action 1 increases with R;

3. relative decision rate, f_1/f_0, increases with R;

4. for fixed rewards u_θ, posterior probabilities of the state 1, $\Pr_{r^*}(\theta = 1 \mid a = 1)$ and $\Pr_{r^*}(\theta = 1 \mid a = 0)$, increase with the prior probability π_1 of the state 1.

[8] Such summary statistic of $\mathcal{P}$ continues to exist when $2 < |X| < \infty$. For any pair of signal realizations (x, x') and an experiment p, let $d_{x,x',p} = \frac{p(x|0)p(x'|1)}{p(x|1)p(x'|0)}$. Then, $\overline{d}$ is the maximum of $d_{x,x',p}$ over all ordered pairs (x, x') and experiments p.

5 Behavioral applications

This section presents four behavioral effects generated by our model. We demonstrate the first three effects—confirmation bias, speed-accuracy complementarity, and overweighting of rare events—in the binary setting from Section 4. The fourth effect—salience of distinct states—will be presented in a setting with multiple states.

5.1 Confirmation bias

Psychologists and economists distinguish at least three mechanisms leading to confirmation bias: (i) People search for evidence selectively, targeting the evidence type in accord with their priors, e.g. Nickerson (1998); (ii) they selectively memorize and recall the data supporting their priors, e.g. Oswald and Grosjean (2004); and (iii) they selectively interpret ambiguous evidence, e.g. Rabin and Schrag (1999) and Fryer et al. (2016). We focus on the first two mechanisms and interpret them in light of our optimal repeated-cognition result.

The next result provides a simple illustration of why some form of confirmation bias is constrained optimal. We consider here an agent who has access to only one symmetric primitive experiment.

Corollary 2. *When action 1 is a priori more attractive, $R \in (1, d)$, and the unique primitive experiment is symmetric, $p(1 \mid 1) = p(0 \mid 0)$, then the agent searches relatively more intensively for signal 1: $\beta_1^* > \beta_0^*$.*

To see how the above result relates to confirmation bias, consider an agent whose task is to announce the realized state of the world: she receives reward $u_1 = u_0 = 1$ if she makes the correct announcement and receives 0 otherwise. The agent finds the state $\theta = 1$ a priori more likely than the state 0, $\pi_1 > \pi_0$.

Consider in this setting the suboptimal decision process that terminates immediately after the first run of the experiment and chooses the action equal to the observed signal: $\beta_0 = \beta_1 = 1$, $\sigma = \sigma_I$. We first observe, paralleling an argument made in subsection 3.3, that such an unbiased process is suboptimal. To see this, assume that the agent has observed the a priori unlikely signal 0. Such a surprised agent is better off by restarting the decision-making instead of terminating with action 0, since if the new run of the process concludes with signal and action 1, then the switch from action 0 to 1 is beneficial. This is because when the experiment p is symmetric, then, conditional on the two conflicting signals, the a priori more common state 1 is relatively more likely. The agent benefits from the second thought whenever she receives the surprising recommendation, and thus will deviate from the uniform search in favor of the a priori likely signal 1.

The optimal strategy resembles the natural process in which the selective memory gives rise to confirmation bias. We consider the fastest optimal strategy, letting $\beta_1^* = 1$. When the agent observes signal 1 that confirms her prior belief, then she terminates and immediately announces the state 1. But if she is surprised, observing signal 0 that contradicts her prior, then she discards the

signal with positive probability β_0^* and repeats the experiment. Although finding the exact optimal value β_0^* may be difficult, the fact that double-checking one's own reasoning when one arrives at a surprising conclusion is a common practice suggests that people are able to deviate from the unbiased suboptimal process in the payoff improving direction.

Let us relate the above effect of confirmation bias to the choice of media outlets. Let each state of the world $\theta \in \{0,1\}$ generate an infinite sequence of signals x_k iid. drawn from the conditional distribution $p(x \mid \theta)$. The agent, whose task is to identify the realized state, can comprehend only one such signal realization and she can access it only via a media outlet. The media outlets differ in their editorial policies $(\beta_x)_x$. Each outlet draws a first signal x_1, terminates and reports x_1 to its readers with probability β_{x_1}, and with the residual probability $1-\beta_{x_1}$ the outlet redraws the new signal x_2, etc, until the outlet terminates its search and reports the last observed signal. The reader of the outlet with an editorial policy β observes signal x in the state θ with probability $s(x \mid \theta; p, \beta) = \frac{\beta_x p(x|\theta)}{\sum_{x'} \beta_{x'} p(x'|\theta)}$.

As in Gentzkow and Shapiro (2006), our agent prefers outlets biased in favor of her prior belief; she prefers an editorial search policy β that is biased in favor of the signal realization that confirms the agent's own prior. For example, a voter a priori favouring, say, Trump, who has time to read only one headline in the media outlet of her choice, will optimally choose an outlet that persistently searches for evidence favorable to Trump, and that will report encountered evidence that is unfavorable to Trump only with a relatively smaller probability.

The source of the media bias in Gentzkow and Shapiro is reputation: Their agents evaluate media outlets confirming their prior beliefs as being relatively reliable information sources. In our case, all outlets are ex ante identical in that they have access to the same signal-generating process and thus reputation does not play a role. Rather, in our case, the demand for the prior-confirming outlets is driven by the information-aggregation friction. When the reader's attention span allows the outlets to report only one signal, then the optimal editorial policy favors the signal that advises the reader correctly in the a priori more likely state, since this state has a large weight in the reader's a priori objective.[9]

5.2 Speed-accuracy complementarity

The binary setting generates the speed-accuracy complementarity effect—a stylized fact stating that delayed choices tend to be less accurate than speedy choices; see the psychology studies of Swensson (1972) and Luce (1986). We establish this effect in the setting from the previous subsection: $u_0 = u_1 = 1$, $\pi_1 > \pi_0$, considering again a symmetric primitive signal distribution, $p(1 \mid 1) = p(0 \mid 0) > 1/2$.

Let $\varphi(\theta, a, t)$ be the joint probability distribution of the state θ, chosen action a, and the reaction time t generated by the solution (p, β^*, σ_I) of the repeated-cognition problem.

Corollary 3. *The probability* $\Pr_\varphi(a = \theta \mid t)$ *of the correct choice decreases with response time* t*.*

[9] See also Calvert (1985), Suen (2004), and Che and Mierendorff (2016) for constrained-optimal media-bias models.

Due to the stationarity of the decision process, the probability of the correct choice conditional on the payoff state is independent of the reaction time: $\Pr_\varphi(a = \theta \mid \theta, t) = \Pr_\varphi(a = \theta \mid \theta)$. At optimum, this conditional probability of the correct choice is larger in the a priori more likely state 1 than in the state 0, reflecting the relative weights of the two states in the a priori objective. Overall, unconditionally on the payoff state, the probability $Pr_\varphi(a = \theta \mid t)$ of the correct choice depends on the response time because t correlates with θ. A long response time indicates that the agent has repeatedly encountered the a priori surprising signal $x = 0$ and has hesitated to terminate. Hence, conditional on large t, the likelihood of the a priori surprising state becomes high. The longer the agent has hesitated, the more likely it is that she is facing the a priori surprising state in which she is making more mistakes.

Corollary 3 is in line with empirical evidence from Ratcliff and McKoon (2008), who asked lab participants to report a binary state visually encoded by a pattern of moving dots on a computer screen. We conceptualize the sequence of the observed dot movements as a sequence of the signals generated by a primitive experiment p in our model. Ratcliff and McKoon's design exhibits a symmetry that justifies the symmetry $p(1 \mid 1) = p(0 \mid 0)$ assumed here. The experiment consisted of two treatments that differed in the prior distribution of the two states. The prior in each treatment was announced to participants in instructions. In line with our predictions, the posterior probability that the participant's announcement is correct is higher when she announces the a priori expected state than when she makes the surprising announcement. The announcements of the surprising state are stochastically delayed relative to the announcements of the a priori expected state. Thus, late announcements are relatively less precise in Ratcliff and McKoon's experimental data in line with Corollary 3.

Such experimental evidence is at odds with the predictions of the influential sequential-learning model of Wald (1945). Wald's agent faces an opportunity cost of time, and learns about a binary state from a sequence of weakly informative signals. The optimal decision procedure terminates once the agent reaches one of the two absorbing posterior beliefs. The optimal absorbing posteriors are independent of the agent's prior (as long as the prior is not too extreme), and when the incentives are symmetric, as in Ratcliff and McKoon's experiment in which participants were simply asked to announce the correct state, the two absorbing posteriors have the same precision.[10] The posterior precision is therefore independent of the response time in Wald's model. These predictions do not match the experimental findings of Ratcliff and McKoon. By contrast, our model generates both the speed-accuracy complementarity, and the empirical monotonicity relationship between the prior and the posteriors (Statement 4 of our Corollary 1).

Fudenberg et al. (2015) have recently proposed a generalization of Wald's model by adding an extra uncertainty about the size of the stake to the sequential-sampling problem. They obtain the speed-accuracy complementarity because their agent interprets long spells of inconclusive learning

[10]See Morris and Strack (2017) for the behavioral equivalence between Wald's problem and the rational inattention problem with a posterior separable cost in binary problems. The independence of the optimal absorbing posteriors from the prior in Wald's problem is equivalent to the analogical independence of the posteriors from the prior in the rational inattention problem in Caplin et al. (2017).

as an indication that the stake is small, thereby leading her to optimally terminate the process sooner with relatively imprecise posteriors. The authors' proposed mechanism differs from ours. While their agent "gives up" after long delays, in our model long delays indicate that the agent has encountered the surprising state in which her decision strategy is less well-adapted. Since there is no uncertainty about the magnitude of the stakes in Ratcliff and McKoon's experiment, the model of Fudenberg et al. does not explain the speed-accuracy complementarity observed in this experiment.

Che and Mierendorff (2016) consider an alternative learning process according to which the agent learns about a binary state from evidence arriving at a Poisson rate. They study the correlation between the accuracy and the response time, and obtain for some parameter specifications that conditional on one particular action being made, the probability that this action is correct declines with time. It should be noted that their result does not aggregate the mistake rates over all possible actions, making the connection to Ratcliff and McKoon less clear.

Ratcliff and McKoon's data also contain evidence suggestive of information aggregation that is abstracted from in the elementary setting studied here. When Ratcliff and McKoon increase the noise in the visual encoding of the state, the response times get longer, which indicates that the participants compensate the poor information source with aggregation of longer signal sequences. We show in Section 6 that our solution of the binary setting and all the related effects extend to agents who partially aggregate information, although we abstract from the trade-off between longer aggregations and the aggregation cost.

5.3 Overweighting of rare events

As in previous subsections, we consider a task consisting of state recognition (i.e. $u(a,\theta) = 1$ if $a = \theta$ and $u(a,\theta) = 0$ otherwise). In contrast to previous applications, we assume that the two states $\theta \in \{0,1\}$ are equally likely, but the distribution of the signal referred to as event in this section $x \in \{0,1\}$ is asymmetric across states. Specifically, the probability of $x = 1$ in the state of the world θ is $\rho_\theta \in (0,1)$ and the probability of $x = 0$ is $1 - \rho_\theta$.

Let us assume, essentially without loss of generality, that $\rho_0 < \rho_1 < 1 - \rho_0$.[11] The a priori probability of event $x = 1$ is $(\rho_0 + \rho_1)/2 < 1/2$, and thus the event $x = 1$ is relatively rarer than $x = 0$. We observe that at the optimum, the agent is relatively more likely to discard the more common event $x = 0$ in agreement with Kahneman and Tversky (1979), who observe that agents tend to overweight rare events.

Corollary 4. *At the optimum, the agent is biased in favor of the rare event $x = 1$: $\beta_1^* > \beta_0^* > 0$ (and her guess of the state equals the observed signal, i.e. $\sigma = \sigma_I$).*

To illustrate the result, consider an agent who is forming her probability belief over a flight accident. The accident probability per flight in the safe state of the world is 10^{-6}, whereas it is 10^{-5} in the dangerous state of the world, and both states are a priori equally likely. The agent can

[11] We can always achieve this by relabeling the states θ and the signals x, unless $\rho_0 = \rho_1$ or $\rho_0 = 1 - \rho_1$.

sequentially observe arbitrarily many past flight outcomes, but cannot aggregate the information, and recalls only the last observed flight. She guesses that the state of the world is dangerous if and only if the last observed flight is eventful.

Consider first an agent who always terminates right after the observation of the first data-point. Such an agent benefits from a "second thought" whenever she observes an uneventful flight: Either the second observed flight will be uneventful, in which case the second thought will have been inconsequential, or the redrawn flight will be eventful and the agent will switch her assessment from the safe to the dangerous state. Such a switch is beneficial since conditional on two contradicting data-points the dangerous state is relatively more likely. More generally, conditional on two contradicting data-points, the state θ for which ρ_θ is closer to $1/2$ is relatively more likely, since it is relatively likely that such a state generates contradicting signals. Thus, relative to the immediate termination strategy, the agent will benefit from discarding the uneventful flight observations with positive probability.

For the assumed accident rates, the optimal ratio of the search intensities is $\beta_1^*/\beta_0^* \approx 316,000$ where the signal $x = 1$ stands for the eventful flight observation. Thus, the agent's search for data is heavily biased towards the rare accidents. This strategy generates probabilities of the correct state identification approximately equal to 0.76 in both states of the world. Since the immediate termination strategy identifies the correct state with a probability equal approximately to half, the bias significantly improves the payoff.

5.4 Salience

Bordalo, Gennaioli, and Shleifer (2012) interpret salience as directed attention focus and quote the popular work by Daniel Kahneman (2011):

> "Our mind has a useful capability to focus on whatever is odd, different or unusual."

The quote states a causal relation between the two features of the salient phenomena: These are (i) odd, different or unusual, and because of (i), people benefit from (ii) focusing their attention on such phenomena. Here, we confirm Kahneman's intuition within our proposed framework. Our microfoundation of the salience effect is related to the insight emerging in psychological research on visual salience. Itti (2007) conceptualizes the visual salience effect as attention allocation to a subset of the visual field that is "sufficiently different from its surroundings to be worthy of [one's] attention."Similarly, in our model, a payoff state is salient if it stands out sufficiently from similar states to be worthy of the focus of the agent's information search.

As before, the agent faces a perceptual task that requires her to announce a random state θ. She is endowed with a primitive perception technology that generates a perceived value θ' of the state. The primitive perception is informative but noisy: The perceived value θ' equals the true state θ with a high probability, but mistakes, $\theta' \neq \theta$, occur sometimes. We view the primitive perception technology as a black-box model of a physiological sensor that generates a noisy first impression θ' of the true state θ. The agent can use the sensor repeatedly but is not able to aggregate the

information. She conditions the repetition of the sensor's use on the most recent perception and announces the terminal perception.

We formalize this perception task as follows. The agent makes an announcement $a \in A = \Theta$, where $2 < |\Theta| < \infty$, and receives payoff $u(a, \theta) = 1$ if her announcement is correct, $a = \theta$, and $u(a, \theta) = 0$ if $a \neq \theta$. The prior is uniform. Each use of the agent's sensor generates a signal/perception $\theta' \in X = \Theta$, with conditional probabilities $p(\theta' \mid \theta)$. We make two assumptions on p:

Symmetry: $p(\theta' \mid \theta) = p(\theta \mid \theta')$.

Sufficient precision: $p(\theta \mid \theta) > p(\theta' \mid \theta)$ for all $\theta \neq \theta'$.

For two states θ_1 and θ_2, we say that θ_1 is *more distinct* than θ_2 if for each state $\theta_3 \neq \theta_1, \theta_2$, $p(\theta_1 \mid \theta_3) < p(\theta_2 \mid \theta_3)$. Suppose for illustration that the perceptual task involves recognition of a color from a set {azure, indigo, red}. Intuitively, the red color stands out of this set, and this is captured by the above definition. Assume that the two shades of blue are similar in that the agent's first impression confuses them in 10% of cases, $p(\text{azure} \mid \text{indigo}) = p(\text{indigo} \mid \text{azure}) = 0.1$, but $p(\theta \mid \text{red}) = p(\text{red} \mid \theta) = 0.01$ for $\theta \in \{\text{azure,indigo}\}$. Then, the red color is more distinct according to our definition than either of the two blue shades.

To simplify the exposition, we assume that the agent uses the identity action strategy σ_I; she announces the state equal to her last perception. We also make a regularity assumption that the optimal termination probabilities β_x are positive for all $x \in \Theta$.[12] Let $r^* = r(p, \beta^*, \sigma_I)$ be the optimal feasible choice rule.

Proposition 3. *If state θ_1 is more distinct than state θ_2, then the agent's terminal perception is biased in favor of the more distinct state θ_1 at the expense of the less distinct state θ_2:*

$$r^*(\theta_1 \mid \theta_2) > r^*(\theta_2 \mid \theta_1).$$

Since the primitive perception technology p is symmetric by assumption, the asymmetry in favor of the distinct state of the optimal terminal perception r^* is driven solely by the optimization of the termination strategy. To gain the intuition for the salience of the distinct states, consider a state θ^* that is similar to many other states and an agent who always terminates the process after the first round: $\beta = \mathbf{1}$. This agent is relatively uninformed whenever she forms perception θ^*, since the true state differs from θ^* with a sizeable probability. The agent with the indistinct perception θ^* would thus benefit from "having a second thought"— i.e., from running the primitive perception formation process once again. The optimal termination strategy involves repeating the primitive process with relatively high probability whenever the agent forms a perception of an indistinct state, and this shifts the terminal perception in favor of the distinct states.

[12] This regularity assumption must be satisfied when $p(\theta \mid \theta)$ is sufficiently close to one for each θ, since then any strategy for which some β_x is zero is dominated by the strategy that immediately terminates and announces the first impression.

6 Robustness checks

We explore here the robustness of our results to an increase in the memory capacity of the agent and to discounting. The first subsection returns to the examples 2 and 3 from Section 2.2 in which the agent is endowed with nontrivial memory and aggregates many observed signals. We show that the two examples are special cases of our baseline model, and hence the optimal rules solving them are second-thought-free. The second subsection accommodates discounting.

6.1 Sophisticated decision processes

Examples 2 and 3 from Section 2.2 seemingly violate our baseline model in that the agent can aggregate information across signal realizations. We show, however, that these examples are formally special cases of our baseline model appropriately specified.

Recall that $\mathcal{P}_{iia}$ is the set of all stochastic choice rules that the agent from example 2 (imperfect information aggregation) can construct. Consider our baseline model with the signal space $X = A$ and the set of the feasible primitive experiments $\mathcal{P} = \mathcal{P}_{iia}$. The set $\mathcal{R}(\mathcal{P}_{iia}) = \{r(p, \beta, \sigma) : p \in \mathcal{P}_{iia}, \beta \in B, \sigma \in S\}$ is then a set of the stochastic choice rules that can be constructed as follows. The agent runs any process $p \in \mathcal{P}_{iia}$, and observes a signal/action recommendation a with probability $p(a \mid \theta)$. She terminates with probability β_a, according to the termination strategy $\beta = (\beta_a)_{a \in A}$, and upon the termination chooses an action $a' = \sigma(a)$, where $\sigma \in S$ is any mapping $A \longrightarrow A$. She reruns the process p with probability $1 - \beta_a$, observes a new action recommendation generated by p, et cetera, until she terminates after a stochastic number of repetitions of the process p.

As it turns out, no new choice rules beyond those from $\mathcal{P}_{iia}$ can be constructed by these selective repetitions. This follows because the repetitions of the rule $p \in \mathcal{P}_{iia}$ with the termination strategy β can always be replicated with an appropriate choice of a different rule in $\mathcal{P}_{iia}$ that whenever p would terminate with a restarts the process from scratch with probability $1 - \beta_a$. Formally:

Lemma 7. $\mathcal{R}(\mathcal{P}_{iia}) = \mathcal{P}_{iia}$.

According to the lemma, example 2 is the special case of our baseline model with $\mathcal{P} = \mathcal{P}_{iia}$, since in such a specification of the baseline model, the set of the feasible rules coincides with those in the example. In particular, the optimal choice rule $p^* \in \mathcal{P}_{iia}$ solving the example coincides with the optimal rule $r^* \in \mathcal{R}(\mathcal{P}_{iia})$ solving this specification of the baseline model.

The repeated-cognition problem with $\mathcal{P} = \mathcal{P}_{iia}$ is purely formal in that the optimal termination probabilities $\beta^*_x = 1$ for all $x \in X = A$, and thus the agent conducts the optimal process $p^* \in \mathcal{P}_{iia}$ only once and terminates. Nevertheless, the observation that p^* solves the repeated-cognition problem has an important implication.

Corollary 5. *The choice rule that solves example 2 (imperfect information aggregation) is second-thought-free.*

Wilson (2014) differs from this example mainly in that she assumes exogenous termination probabilities. By adding optimization over the terminations to the model of Wilson, we gained

the partial characterization of the optimal choice rule without fully solving the problem: One can conclude that the optimal choice rule is second-thought-free without analyzing the optimal use of the memory states.

The same argument applies to example 3 (partial forgetting). As with the previous example, let $\mathcal{R}(\mathcal{P}_{pf})$ be the set of the feasible choice rules in our baseline model with the set of the feasible primitive experiments $\mathcal{P}$ identified with $\mathcal{P}_{pf}$.

Lemma 8. $\mathcal{R}(\mathcal{P}_{pf}) = \mathcal{P}_{pf}$.

Thus, again, the rule $p^* \in \mathcal{P}_{pf}$ solving example 3, and the optimal rule $r^* \in \mathcal{R}(\mathcal{P}_{pf})$ coincide, and thus the rule solving the example must be second-thought-free.

Corollary 6. *The choice rule that solves example 3 (partial forgetting) is second-thought-free.*

Importantly, when the action and state sets are binary (while the signal space can be arbitrary), then Proposition 2 derived above for memoryless agents applies to the sophisticated agents from examples 2 and 3. The optimal choice rules solving the examples simply satisfy Proposition 2 with $\overline{d}$ set to the maximal perceptual distance among the rules p in P_{iia} and $\mathcal{P}_{pf}$, respectively.

6.2 Costly delay

Our baseline model abstracts from the cost of time in that the agent is only concerned with how the repetitions of the signal extraction affect the correlation of the signal with the state. We now incorporate discounting.

We continue to study the baseline model from Section 2.1, except that the agent discounts future payoffs exponentially with the discount factor $\delta \in (0, 1)$. To accommodate discounting, we redefine the choice rule induced by the experiment p, the termination strategy β and the action strategy σ as follows.

$$r_\delta(a \mid \theta; p, \beta, \sigma) = \sum_t \sum_{\mathbf{x}^t : \sigma(x_t) = a} \delta^t \rho\left(\mathbf{x}^t \mid \theta; p, \beta\right), \tag{11}$$

where $\rho\left(\mathbf{x}^t \mid \theta; p, \beta\right)$ defined in (1) is the conditional probability of the signal history $\mathbf{x}^t$. That is, $r_\delta(a \mid \theta; p, \beta, \sigma)$ is the *discounted* probability of the choice of action a in the state θ. When $\delta = 1$, then (11) coincides with our baseline definition of the choice rule $r(p, \beta, \sigma)$.

The set of the feasible discounted rules is $\mathcal{R}_\delta(\mathcal{P}) = \{r_\delta(p, \beta, \sigma) : p \in \mathcal{P}, \beta \in B, \sigma \in S\}$. The discounted repeated-cognition problem is to select a feasible rule r_δ that maximizes the expected payoff:

$$\max_{r_\delta \in \mathcal{R}_\delta(\mathcal{P})} \sum_{\theta \in \Theta, a \in A} \pi_\theta r_\delta(a \mid \theta) u(a, \theta), \tag{12}$$

where discounting is incorporated in the definition of the feasible rules.

The next result generalizes the second-thought-free condition. Let $r^*_\delta = r_\delta(p^*, \beta^*, \sigma^*)$ be the choice rule solving the discounted repeated-cognition problem (12).

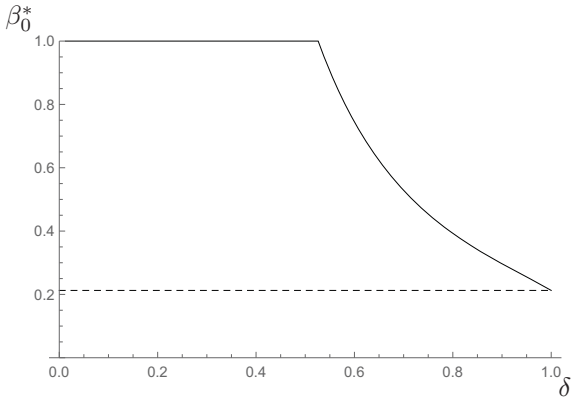

Figure 3: Confirmation bias with discounting. Action 1 is a priori more attractive: $\pi_1 u_1 = 10 \times \pi_0 u_0$. The primitive experiment is symmetric: $p(1 \mid 1) = p(0 \mid 0) = .9$. The agent terminates immediately when she receives signal 1, $\beta_1^* = 1$. When $\delta > .53$, then the agent is biased towards state 1: when she encounters signal 0, then she terminates the decision-process with a probability only $\beta_0^*(\delta) < 1$ (the full curve). The dotted line is β_0^*/β_1^* from the baseline model without discounting.

Proposition 4. *If the termination strategy $\beta_x^* \in (0,1)$ is interior for all x such that $\sigma^*(x) = a$, then*

$$\sum_{\theta \in \Theta} \pi_\theta u(a,\theta) r_\delta^*(a \mid \theta) = \delta \sum_{\theta \in \Theta, a' \in A} \pi_\theta u(a',\theta) r_\delta^*(a' \mid \theta) r_\delta^*(a \mid \theta). \tag{13}$$

The condition has the same interpretation as the second-thought-free condition in the absence of discounting. The left-hand side is the payoff for following the optimal decision process r_δ^* summed up across all contingencies that terminate with choice of a. The right-hand side is the payoff that the agent would get across the same contingencies if she restarted the decision process r_δ^* instead of the termination.

For illustration, we now revisit the confirmation bias application from Section 5.1 with an impatient agent. We will find that, unless discounting is too strong, the impatient agent chooses qualitatively the same strategy as the patient one, although the impatient agent speeds up her decision-making by choosing larger termination probabilities.

The setting is as follows. The agent chooses $a \in \{0,1\}$ and receives $u(a,\theta) = u_\theta > 0$ if $a = \theta$, and zero reward otherwise. Action 1 is a priori more attractive than action 0; $\pi_1 u_1 > \pi_0 u_0$. The agent has access to a single primitive experiment p that generates signals in $X = \{0,1\}$. The experiment is symmetric with probabilities $p(1 \mid 1) = p(0 \mid 0) = \alpha > 1/2$.

Proposition 5. *The agent chooses the action equal to the last observed signal. She terminates her decision-making immediately after she encounters signal $x = 1$: $\beta_1^* = 1$. When $\delta \in \left(\frac{1}{\alpha+(1-\alpha)R}, 1\right]$, then the agent who receives signal $x = 0$ terminates with interior probability $\beta_0^* \in (0,1)$ that decreases in δ. When $\delta \in \left(0, \frac{1}{\alpha+(1-\alpha)R}\right)$, then the agent terminates immediately: $\beta_0^* = \beta_1^* = 1$.*

6.3 Discussion

The second-thought-free condition is related to Piccione and Rubinstein (1997), who show that the ex ante optimal decision strategy of a forgetful decision-maker can be thought of as a team equilibrium of multiple selves, with each self representing the decision-maker at an information set. Each self takes the strategies of the other selves as given, internalizes the other selves' strategies in her Bayesian inference, and maximizes the decision-maker's payoff. Our agent is forgetful in that she conditions terminations only on the last signal, not on the signal history. As in Piccione and Rubinstein, the optimal termination strategy is a team equilibrium in the sense of the modified multi-self equilibrium defined there. The self who has received a signal makes inferences about the state both from the the observed signal and from the fact that the previous selves (if any) have not terminated. Given her equilibrium posterior belief, each self decides whether to terminate with the action that is optimal under the formed posterior or to delegate the decision to the next self. The equilibrium is mixed, with each self indifferent between terminating and passing the decision forward.

Meyer (1991) studies optimal biases in a sequential-learning problem of an agent who receives a sequence of binary signals and, unlike our agent, can aggregate the sequence of signals. She analyzes the best signal structure maintaining that the signal realization is binary. Her main insight is that some asymmetries in the signal structure are optimal. This can easily be understood in a two-period version, as symmetry of the conditional signal distributions would imply that the second signal is worthless. Although the observation that asymmetries in the information structure may be optimal arises both in her and our framework, the two papers study distinct optimizations. While our agent controls termination probabilities in a stationary decision process, Meyer's agent controls the choice of a Blackwell experiment in each round of a non-stationary process.

In the above setting, we have assumed that the agent chooses optimally the action strategy $\sigma : X \to A$. In some contexts, the action strategy may instead be hardwired by automatic responses and possibly suboptimal. We note that as long as the agent can optimally adjust the termination strategy β, the second-thought-free condition continues to hold since it follows from the first-order condition with respect to the termination strategy only. Behavioral insights such as those highlighted in Section 5 would be unaltered in such environments.

7 Summary

Agents, who cannot comprehend all facts available to them, benefit from selective attention. We show that agents can implement a targeted information search in a process that resembles the natural phenomenon of hesitation. Like a hesitant person, the agent can, conditional on the action contemplated, decide whether she implements the action or whether she will have a second thought, and run the cognition process once more. Such hesitation can be productive, despite consisting of repetitions of the same stochastic cognition process. By conditioning the probability of the repetition on the conclusion of the reasoning, the agent controls the correlation of her terminal

conclusion and the payoff state. The optimal decision process arising in our model exhibits natural hesitation patterns: The agent will have second thoughts—that is, she will repeat her cognition—whenever the expected payoff for the currently favored choice is inferior to the expected payoff for continuing decision-making. At optimum, the agent terminating the decision-making must be indifferent between terminating with the currently contemplated action, and repeating the process. In a sense, the condition formalizes the concept of a reasonable doubt. Abstracting from many considerations such as information aggregation, a jury deciding a trial under common law should be, if using the optimal decision procedure, indifferent between declaring a verdict and announcing a hung jury and initiate retrial.

Let us conclude by reviewing the limitations of our main result. The central assumption—the ability of the agent to freely repeat her decision process—may fail for several reasons. One reason is that the agent may only have access to a limited data set that constrains her to a finite number of repetitions of the primitive decision process, making the optimal termination strategy non-stationary. Another complication arises if the outcomes of distinct runs of the same cognition process are not conditionally independent as assumed in our model; this may arise if some cognition errors are systematic and are likely to emerge in distinct repetitions of the cognition. We conjecture that the second-thought-free condition holds in such a case, with the agent internalizing the correlations between the cognition runs.

A Proofs

A.1 Proofs for Section 3

Proof of Lemma 1. Suppose by contradiction that (4) holds with strict inequality for some a chosen with positive probability. Then, $\mathrm{E}_\alpha\left[\mathrm{E}_\alpha\left[u(a_1,\theta)\mid a_1\right]\right] > \mathrm{E}_\alpha\left[\mathrm{E}_\alpha\left[u(a_2,\theta)\mid a_1\right]\right]$, and applying the law of iterated expectation, this simplifies to $\mathrm{E}_\alpha\left[u(a_1,\theta)\right] > \mathrm{E}_\alpha\left[u(a_2,\theta)\right]$. This establishes the contradiction since a_1 and a_2 are conditionally iid. draws generated by the rule r. □

Proof of Lemma 2. The effective experiment $s(p,\beta)$ satisfies a recursion

$$s(x\mid\theta;p,\beta)=\beta_x p(x\mid\theta)+\sum_{x'\in X}\left(1-\beta_{x'}\right)p\left(x'\mid\theta\right)s(x\mid\theta;p,\beta),$$

where the first summand is the probability that the agent terminates with signal x after the first run of the primitive experiment p, and the second summand is the probability that the agent continues with decision-making after the first run of p and terminates with x later. Solving for $s(x\mid\theta;p,\beta)$ gives (6). □

Proof of Proposition 1. Let (p^*,β^*,σ^*) solve the repeated-cognition problem. Consider an action a chosen with a positive probability. There must exist x such that $\sigma^*(x)=a$ and $\beta^*_x>0$. Therefore, the constraint $\beta_x\geq 0$ is not binding for this x, and the first-order condition of problem (7) with

respect to β_x is:

$$\begin{aligned}\sum_{\theta\in\Theta}\pi_\theta\frac{s\left(x\mid\theta;p^*,\beta^*\right)}{\beta_x^*}u(a,\theta)-\sum_{\theta\in\Theta,x'\in X}\pi_\theta s\left(x'\mid\theta;p^*,\beta^*\right)\frac{s\left(x\mid\theta;p^*,\beta^*\right)}{\beta_x^*}u\left(\sigma^*(x'),\theta\right) &= \\ \sum_{\theta\in\Theta}\pi_\theta\frac{s\left(x\mid\theta;p^*,\beta^*\right)}{\beta_x^*}u(a,\theta)-\sum_{\theta\in\Theta,a'\in A}\pi_\theta r\left(a'\mid\theta;p^*,\beta^*,\sigma^*\right)\frac{s\left(x\mid\theta;p^*,\beta^*\right)}{\beta_x^*}u\left(a',\theta\right) &\geq 0,\end{aligned}$$

where we have summed over all x' such that $\sigma^*(x')=a'$ in the second line. Multiplication by β_x^* and summation over all x such that $\sigma^*(x)=a$ and $\beta_x>0$ give

$$\sum_{\theta\in\Theta}\pi_\theta r\left(a\mid\theta;p^*,\beta^*,\sigma^*\right)u(a,\theta)-\sum_{\theta\in\Theta,a'\in A}\pi_\theta r\left(a'\mid\theta;p^*,\beta^*,\sigma^*\right)r\left(a\mid\theta;p^*,\beta^*,\sigma^*\right)u(a',\theta)\geq 0.$$

Division of this by $\sum_\theta\pi_\theta r\left(a\mid\theta;p^*,\beta^*,\sigma^*\right)$ leads to (4). Lemma 1 implies (5). □

A.2 Proofs for Section 4

Proof of Lemma 3. Assume that there exists a solution with β_x positive for $n>2$ signals $x\in X$. We show that then there exists a solution with $n-1$ positive signals. The proposition follows from the induction on n.

Let us prove the induction step. Fix the primitive experiment p employed by the agent, let β be an optimal termination strategy for the given p, and let X' be the set of signals with positive β_x, and write shortly $s(x\mid\theta)$ for the effective experiment $s(x\mid\theta;p,\beta)$ induced by p and β. Let us abuse notation by letting $s(x)=\sum_\theta\pi_\theta s(x\mid\theta)$ stand for the unconditional effective probability of x. For $x\in X'$ let $q_x\in\Delta(\Theta)$ be the posterior belief upon terminating with x: $q_x(\theta)=\pi_\theta s(x\mid\theta)/s(x)$.

Since $|X'|>2$ and the state space Θ is binary, there exists a signal $x^*\in X'$ such that q_{x^*} is in the convex hull of the posteriors q_x, $x\in X'\setminus\{x^*\}$. Let μ_x be the coefficients that decompose q_{x^*} into q_x, $x\in X'\setminus\{x^*\}$. That is, $\mu\in\Delta\left(X'\setminus\{x^*\}\right)$ such that $q_{x^*}(\theta)=\sum_{x\in X'\setminus\{x^*\}}\mu_x q_x(\theta)$ for all $\theta\in\Theta$.

We will construct an alternative feasible effective experiment $\tilde{s}(x\mid\theta)$ with unconditional probabilities of x denoted by $\tilde{s}(x)$ and the posteriors $\pi_\theta\tilde{s}(x\mid\theta)/\tilde{s}(x)$ denoted by $\tilde{q}_x(\theta)$ such that:

$$\tilde{s}(x)=\begin{cases}s(x)+s(x^*)\mu_x \text{ if } x\in X'\setminus\{x^*\},\\ 0 \text{ otherwise,}\end{cases}\tag{14}$$

and

$$\tilde{q}_x(\theta)=q_x(\theta) \text{ for all } x\in X'\setminus\{x^*\},\theta\in\Theta.\tag{15}$$

Since the experiment $\tilde{s}$ is more informative than s (in the sense of the Blackwell comparison), there exists a solution with this alternative feasible effective experiment $\tilde{s}$, as needed for the induction step.

It remains to construct $\tilde{s}$. Note that if an effective experiment $s(x\mid\theta;p,\beta)=\frac{\beta_x p(x|\theta)}{\sum_{x'}\beta_{x'}p(x'|\theta)}$ is

induced by some p and β, then for any vector of probabilities $\tilde{\beta}_x$, the experiment

$$\tilde{s}(x \mid \theta) = \frac{\tilde{\beta}_x s(x \mid \theta; p, \beta)}{\sum_{x' \in X} \tilde{\beta}_{x'} s(x' \mid \theta; p, \beta)} = \frac{\tilde{\beta}_x \beta_x p(x \mid \theta)}{\sum_{x' \in X} \tilde{\beta}_{x'} \beta_{x'} p(x' \mid \theta)}$$

is also feasible, since it is induced by p and $\beta' = (\tilde{\beta}_x \beta_x)_{x \in X}$.

We claim that if

$$\tilde{\beta}_x = \begin{cases} c\left(1 + \frac{s(x^*)\mu_x}{s(x)}\right) & \text{if } x \in X' \setminus \{x^*\}, \\ 0 \text{ otherwise,} \end{cases}$$

where c is a constant such that $\tilde{\beta}_x \in (0,1)$ for all $x \in X$, then the resulting $\tilde{s}$ satisfies the properties (14) and (15). Let us check:

$$\begin{aligned}
\tilde{s}(x \mid \theta) &= \frac{\tilde{\beta}_x s(x \mid \theta)}{\sum_{x' \in X' \setminus \{x^*\}} \tilde{\beta}_{x'} s(x' \mid \theta)} \\
&= \frac{\tilde{\beta}_x s(x \mid \theta)}{c\left(\sum_{x' \in X' \setminus \{x^*\}} s(x' \mid \theta) + \sum_{x' \in X' \setminus \{x^*\}} \frac{s(x^*)\mu_{x'}}{s(x')} s(x' \mid \theta)\right)} \\
&= \frac{\tilde{\beta}_x s(x \mid \theta)}{c\left(\sum_{x' \in X' \setminus \{x^*\}} s(x' \mid \theta) + \sum_{x' \in X' \setminus \{x^*\}} \frac{s(x^*)\mu_{x'}}{\pi_\theta} q_{x'}(\theta)\right)} \\
&= \frac{\tilde{\beta}_x s(x \mid \theta)}{c\left(\sum_{x' \in X' \setminus \{x^*\}} s(x' \mid \theta) + \frac{s(x^*)}{\pi_\theta} q_{x^*}(\theta)\right)} \\
&= \frac{\tilde{\beta}_x s(x \mid \theta)}{c\left(\sum_{x' \in X' \setminus \{x^*\}} s(x' \mid \theta) + s(x^* \mid \theta)\right)} \\
&= \frac{\tilde{\beta}_x s(x \mid \theta)}{c} \\
&= \left(1 + \frac{s(x^*)\mu_x}{s(x)}\right) s(x \mid \theta).
\end{aligned}$$

The property (14) holds since for all $x \in X' \setminus \{x^*\}$:

$$\tilde{s}(x) = \left(1 + \frac{s(x^*)\mu_x}{s(x)}\right) s(x) = s(x) + s(x^*)\mu_x.$$

To establish the property (15), check that for all $x \in X' \setminus \{x^*\}$ and all $\theta \in \Theta$:

$$\tilde{q}_x(\theta) = \frac{\pi_\theta \tilde{s}(x \mid \theta)}{\sum_{\theta' \in \Theta} \tilde{s}(x \mid \theta')} = \frac{\pi_\theta \left(1 + \frac{s(x^*)\mu_x}{s(x)}\right) s(x \mid \theta)}{\sum_{\theta' \in \Theta} \left(1 + \frac{s(x^*)\mu_x}{s(x)}\right) s(x \mid \theta')} = \frac{\pi_\theta s(x \mid \theta)}{\sum_{\theta' \in \Theta} s(x \mid \theta')} = q_x(\theta).$$

□

Proof of Lemma 4. For any positive β,

$$\frac{r(1 \mid 1; p, \beta, \sigma_I) r(0 \mid 0; p, \beta, \sigma_I)}{r(0 \mid 1; p, \beta, \sigma_I) r(1 \mid 0; p, \beta, \sigma_I)} = \frac{\frac{\beta_1 p(1|1)}{\sum_x \beta_x p(x|1)} \frac{\beta_0 p(0|0)}{\sum_x \beta_x p(x|0)}}{\frac{\beta_0 p(0|1)}{\sum_x \beta_x p(x|1)} \frac{\beta_1 p(1|0)}{\sum_x \beta_x p(x|0)}} = \frac{p(1 \mid 1) p(0 \mid 0)}{p(0 \mid 1) p(1 \mid 0)} = d_p.$$

Thus, every $r \in \mathcal{R}_{p,\sigma_I}$ either always selects a same action, or satisfies $\frac{r(1|1)r(0|0)}{r(0|1)r(0|1)} = d_p$. Vice versa, if a rule r' satisfies $\frac{r'(1|1)r'(0|0)}{r'(0|1)r'(0|1)} = d_p$, then it belongs to $\mathcal{R}_{p,\sigma_I}$. To see this, let r_a denote the rule that always selects action a. Consider positive β_0, and note that $r\left(p, (\beta_0, \beta_1), \sigma_I\right)$ is continuous in β_1, and converges to r_1 and r_0 as β_1 approaches 1 and 0. Thus, there exists β such that $r'(1 \mid 1) = r\left(1 \mid 1; p, \beta, \sigma_I\right)$. Moreover, there is a unique rule $\tilde{r}$ that satisfies $\tilde{r}(1 \mid 1) = r'(1 \mid 1)$ and $\frac{\tilde{r}(1|1)\tilde{r}(0|0)}{\tilde{r}(0|1)\tilde{r}(0|1)} = d_p$. Thus, r' must be $r\left(p, \beta, \sigma_I\right)$ and hence constructible from p.[13] □

Proof of Lemma 5. The statement is trivial when $r(p, \beta, \sigma)$ chooses an action a' with probability 1, since then we can set $\beta'_{a'} = 1$ and $\beta'_x = 0$ for $x \neq a'$. Accordingly, assume that both actions are chosen with positive probabilities under the rule $r(p, \beta, \sigma)$ and $\sigma(x) = 1 - x$. For the sake of contradiction, assume that $r(p, \beta, \sigma)$ achieves a higher payoff than all rules constructible with p and σ_I. Then, the payoff difference between the rule $r(p, \beta, \sigma)$ and the choice rule that always selects $a = 1$ must be positive:

$$\begin{aligned}
\pi_0 u_0 r(0 \mid 0; p, \beta, \sigma) + \pi_1 u_1 r(1 \mid 1; p, \beta, \sigma) - \pi_1 u_1 &= \\
\pi_0 u_0 r\left(1 \mid 0; p, \beta, \sigma_I\right) + \pi_1 u_1 r\left(0 \mid 1; p, \beta, \sigma_I\right) - \pi_1 u_1 &= \\
\pi_0 u_0 r\left(1 \mid 0; p, \beta, \sigma_I\right) - \pi_1 u_1 r\left(1 \mid 1; p, \beta, \sigma_I\right) &> 0,
\end{aligned}$$

where we have used $r(a \mid \theta; p, \beta, \sigma_I) = r(1 - a \mid \theta; p, \beta, \sigma)$ for the first equality. Similarly, the payoff difference between the rule $r(p, \beta, \sigma)$ and the rule that always selects $a = 0$ must be positive:

$$\begin{aligned}
\pi_0 u_0 r(0 \mid 0; p, \beta, \sigma) + \pi_1 u_1 r(1 \mid 1; p, \beta, \sigma) - \pi_0 u_0 &= \\
\pi_0 u_0 r\left(1 \mid 0; p, \beta, \sigma_I\right) + \pi_1 u_1 r\left(0 \mid 1; p, \beta, \sigma_I\right) - \pi_0 u_0 &= \\
\pi_1 u_1 r(0 \mid 1; p, \beta, \sigma_I) - \pi_0 u_0 r(0 \mid 0; p, \beta, \sigma_I) &> 0.
\end{aligned}$$

The last two inequalities imply

$$\frac{r(1 \mid 1; p, \beta, \sigma_I)}{r(1 \mid 0; p, \beta, \sigma_I)} < \frac{\pi_0 u_0}{\pi_1 u_1} < \frac{r(0 \mid 1; p, \beta, \sigma_I)}{r(0 \mid 0; p, \beta, \sigma_I)},$$

which establishes contradiction because by Lemma 4, the rule $r(a \mid \theta; p, \beta, \sigma_I)$ satisfies

$$\frac{r(1 \mid 1; p, \beta, \sigma_I) r(0 \mid 0; p, \beta, \sigma_I)}{r(1 \mid 0; p, \beta, \sigma_I) r(0 \mid 1; p, \beta, \sigma_I)} = \frac{p(1 \mid 1) p(0 \mid 0)}{p(1 \mid 0) p(0 \mid 1)},$$

[13] Rules r_a that always select an action a can be trivially constructed from p and σ_I by using $\beta_a = 1$ and $\beta_x = 0$ for $x \neq a$.

and therefore it inherits the monotone likelihood ratio property from p. □

Proof of Lemma 6. Consider the choice rule $r(p, \beta, \sigma_I)$ constructed from the experiment p with perceptual distance $d_p = d$, and fix the probability $r(0 \mid 0; p, \beta, \sigma_I) = \alpha$ of the correct choice in state 0 to a value $\alpha \in (0, 1)$. Then, by Lemma 4, the probability $r(1 \mid 1; p, \beta, \sigma_I)$ of the correct choice in state 1 satisfies

$$\frac{r(1 \mid 1; p, \beta, \sigma_I)\alpha}{(1 - r(1 \mid 1; p, \beta, \sigma_I))(1 - \alpha)} = d.$$

For each α, the solution for $r(1 \mid 1; p, \beta, \sigma_I)$ of this equation increases in d. □

Proof of Proposition 2. The agent's objective is linear with respect to the choice rule $r(p, \beta, \sigma)$. Thus, the optimal rule is the point of tangency of the set $\mathcal{R}_{\overline{p},\sigma_I}$ of the feasible rules and of an indifference line; see Figure 2. The slope $\frac{dr(0|0;\overline{p},\beta,\sigma_I)}{dr(1|1;\overline{p},\beta,\sigma_I)}$ is decreasing in $r\left(1 \mid 1; \overline{p}, \beta, \sigma_I\right)$ and attains value $-1/\overline{d}$ for $r\left(1 \mid 1; \overline{p}, \beta, \sigma_I\right) = 0$, and value $-\overline{d}$ for $r\left(1 \mid 1; \overline{p}, \beta, \sigma_I\right) = 1$. Thus, when $R < 1/\overline{d}$ or $R > \overline{d}$, then the problem has the corner solution as specified in statements 1 and 2 of the proposition.

When $R \in \left(1/\overline{d}, \overline{d}\right)$, then the optimal choice rule $r^* = r\left(\overline{p}, \beta^*, \sigma_I\right)$ satisfies the feasibility condition $\frac{r^*(1|1)r^*(0|0)}{r^*(0|1)r^*(0|1)} = \overline{d}$, the second-thought-free condition (5) (applied to action $a = 1$):

$$\pi_1 u_1 r^*(1 \mid 1) = \pi_0 u_0 r^*(0 \mid 0) r^*(1 \mid 0) + \pi_1 u_1 r^*(1 \mid 1) r^*(1 \mid 1),$$

and two normalization conditions $\sum_a r^*(a \mid \theta) = 1$, for $\theta \in \{0, 1\}$. These four conditions jointly imply the explicit solution for the optimal choice rule in (9). The expression (10) for β_1^*/β_0^* follows from (9) and the condition $\frac{r^*(1|\theta)}{r^*(0|\theta)} = \frac{\beta_1^* \overline{p}(1|\theta)}{\beta_0^* \overline{p}(0|\theta)}$. □

Proof of Corollary 1. The comparative statics results 1 and 2 follow from the signs of the derivatives of the explicit solutions (9) and (10). For the relative response rate in statement 3, the monotone likelihood property of $\overline{p}$ implies that

$$\frac{f_1}{f_0} = \frac{\beta_0^* \overline{p}(0 \mid 1) + \beta_1^* \overline{p}(1 \mid 1)}{\beta_0^* \overline{p}(0 \mid 0) + \beta_1^* \overline{p}(1 \mid 0)}$$

increases in β_1^*/β_0^*, since

$$\frac{d\frac{f_1}{f_0}}{d\frac{\beta_1^*}{\beta_0^*}} = \frac{\overline{p}(1 \mid 1)\overline{p}(0 \mid 0) - \overline{p}(0 \mid 1)\overline{p}(1 \mid 0)}{\left(\overline{p}(0 \mid 0) + \frac{\beta_1^*}{\beta_0^*}\overline{p}(1 \mid 0)\right)^2} > 0.$$

For statement 4, note that the choice rule r^* characterized in (9) is homogeneous with degree zero with respect to u_0 and u_1. Thus, without loss of generality, we can set $u_0 = 1$. Bayes rule

implies

$$\begin{aligned}\Pr_{r^*}(\theta=1\mid a=1) &= r^*(1\mid 1)\frac{\pi_1}{\pi_1 r^*(1\mid 1)+(1-\pi_1)\left(1-r^*(0\mid 0)\right)},\\ \Pr_{r^*}(\theta=1\mid a=0) &= \left(1-r^*(1\mid 1)\right)\frac{\pi_1}{\pi_1\left(1-r^*(1\mid 1)\right)+(1-\pi_1)r^*(0\mid 0)}.\end{aligned}$$

We substitute the expressions for $r^*(0\mid 0)$ and $r^*(1\mid 1)$ from (9) and take derivatives with respect to π_1 to get

$$\begin{aligned}\frac{d}{d\pi_1}\Pr_{r^*}(\theta=1\mid a=1) &= \frac{\overline{d}\sqrt{\frac{\overline{d}\pi_1 u_1}{1-\pi_1}}}{2\left(\overline{d}\pi_1+(1-\pi_1)\sqrt{\frac{\overline{d}\pi_1 u_1}{1-\pi_1}}\right)^2},\\ \frac{d}{d\pi_1}\Pr_{r^*}(\theta=1\mid a=0) &= \frac{\sqrt{\frac{\overline{d}\pi_1 u_1}{1-\pi_1}}}{2\left(\pi_1+(1-\pi_1)\sqrt{\frac{\overline{d}\pi_1 u_1}{1-\pi_1}}\right)^2},\end{aligned}$$

where both derivatives are positive, as needed. □

A.3 Proofs for Section 5

Proof of Corollary 2. Since β_1^*/β_0^* increases in R, it suffices to show that $\beta_1^*/\beta_0^*=1$ when $R=1$ and the primitive experiment p is symmetric. Indeed, when $R=1$, then by (10),

$$\frac{\beta_1^*}{\beta_0^*}=\sqrt{d_p}\frac{p(0\mid 1)}{p(1\mid 1)}=\sqrt{\frac{p(0\mid 0)p(0\mid 1)}{p(1\mid 1)p(1\mid 0)}}=1,$$

where the last equality follows from the symmetry of p. □

Proof of Corollary 3. $\beta_1^*>\beta_0^*$ by Corollary (2) since $\pi_1u_1>\pi_0u_0$. Recall that $f_\theta=\beta_1^*p(1\mid\theta)+\beta_0^*p(0\mid\theta)$ denotes the probability of termination per each round in state θ, and that the response time t in the state θ is geometrically distributed with the decision rate f_θ: $\Pr_\varphi(t\mid\theta)=f_\theta(1-f_\theta)^t$ for $t=0,1,\ldots$. Since $p(1\mid 1)=p(0\mid 0)>p(1\mid 0)=p(1\mid 0)$ and $\beta_1^*>\beta_0^*$, the decision rate is higher in state 1 than in state 0: $f_1>f_0$. Thus, the likelihood ratio $\Pr_\varphi(t\mid\theta=1)/\Pr_\varphi(t\mid\theta=0)$ decreases with t, and hence $\Pr_\varphi(\theta=1\mid t)$ decreases in t. The fact that $\beta_1^*>\beta_0^*$, and the symmetry of p implies that the probability of the correct choice is larger in state 1 than in state 0:

$$r\left(1\mid 1;p,\beta^*,\sigma_I\right)=\frac{\beta_1^*p(1\mid 1)}{\beta_0^*p(0\mid 1)+\beta_1^*p(1\mid 1)}>\frac{\beta_0^*p(0\mid 0)}{\beta_0^*p(0\mid 0)+\beta_1^*p(1\mid 0)}=r\left(0\mid 0;p,\beta^*,\sigma_I\right).$$

Since $\Pr_\varphi(a=\theta\mid t)=\Pr_\varphi(\theta=1\mid t)r(1\mid 1;p,\beta^*,\sigma_I)+\Pr_\varphi(\theta=0\mid t)r(0\mid 0;p,\beta^*,\sigma_I)$, the result obtains. □

Proof of Corollary 4. The belief formation problem studied is a special case of our binary setting

with the primitive experiment $p(x \mid \theta) = \rho_\theta$ if $x = 1$, $p(x \mid \theta) = 1 - \rho_\theta$ if $x = 0$ and with equally a priori attractive actions, $R = 1$. Since $\rho_0 < \rho_1$, the setting satisfies the monotone likelihood property, and thus by Lemma 5, there exists a solution with $\sigma(x) = x$. Since $R = 1 \in (1/d, d)$, Proposition 2 implies that the agent's behavior is stochastic, both β_0^* and β_1^* are positive, and the ratio of the search intensities β_1^*/β_0^* satisfies (10). Since $R = 1$, (10) simplifies to

$$\frac{\beta_1^*}{\beta_0^*} = d_p^{1/2} \frac{p(0 \mid 1)}{p(1 \mid 1)} = \left(\frac{p(0 \mid 1) p(0 \mid 0)}{p(1 \mid 1) p(1 \mid 0)} \right)^{1/2} = \left(\frac{(1 - \rho_1)(1 - \rho_0)}{\rho_1 \rho_0} \right)^{1/2}.$$

The inequality $\beta_1^* > \beta_0^*$ follows from $\rho_0 < \rho_1 < 1 - \rho_0$. □

The next result is an auxiliary lemma used in the proof of Proposition 3.

Lemma 9. *Suppose that termination probabilities β_x are positive for all $x \in \Theta$. Then, the optimal effective choice rule r^* satisfies for any pair of states θ, $\theta' \in \Theta$:*

$$r^*(\theta \mid \theta) r^*(\theta' \mid \theta) = r^*(\theta \mid \theta') r^*(\theta' \mid \theta'). \tag{16}$$

Condition (16) is a strengthening of the second-thought-free condition (5). It requires that the agent who has terminated the decision process with perception θ, and knows that the second run of the process r^* terminates with a value θ' is indifferent between θ and θ'. This condition is stronger than the second-thought-free condition (5), since (5) requires (16) to hold only on average across all θ'. This strengthening holds for the special case of this application with a symmetric experiment p.

Proof of Lemma 9. The optimal effective choice rule satisfies the second-thought-free condition (5), equivalent to:

$$r^*(\theta \mid \theta) = \sum_{\theta' \in \Theta} r^*(\theta \mid \theta') r^*(\theta' \mid \theta') \text{ for all } \theta \in \Theta,$$

which after two algebraic steps gives:

$$r^*(\theta \mid \theta)\big(1 - r^*(\theta \mid \theta)\big) = \sum_{\theta' \neq \theta} r^*(\theta \mid \theta') r^*(\theta' \mid \theta') \text{ for all } \theta \in \Theta,$$

$$\sum_{\theta' \neq \theta} r^*(\theta \mid \theta) r^*(\theta' \mid \theta) = \sum_{\theta' \neq \theta} r^*(\theta' \mid \theta') r^*(\theta \mid \theta') \text{ for all } \theta \in \Theta.$$

The last system of equations is formally equivalent to the system of balance conditions for a Markov chain. To see this, consider an ergodic Markov chain with transition probabilities from θ to θ' equal to $r^*(\theta' \mid \theta)$. The balance condition for the stationary distribution $\mu(\theta)$ of this chain is

$$\sum_{\theta' \neq \theta} \mu(\theta) r^*(\theta' \mid \theta) = \sum_{\theta' \neq \theta} \mu(\theta') r^*(\theta \mid \theta') \text{ for all } \theta \in \Theta,$$

and thus $r^*(\theta \mid \theta)$ is proportional to the ergodic probability $\mu(\theta)$ of the state θ for the chain with transition probabilities $r^*(\theta' \mid \theta)$.

Recall that if a Markov chain with transition probabilities $m(\theta' \mid \theta)$ is reversible, then its stationary distribution $\mu(\theta)$ satisfies detailed balance conditions

$$\mu(\theta)m(\theta' \mid \theta) = \mu(\theta')m(\theta \mid \theta') \text{ for all } \theta \neq \theta'.$$

Thus, it suffices to prove that the probabilities $r^*(\theta' \mid \theta)$ constitute a reversible Markov chain.

Recall that a Markov chain $m(\theta' \mid \theta)$ is reversible if and only if it satisfies the Kolmogorov criterion, which requires for all sequences of states $\theta_1, \theta_2, \ldots, \theta_n$,

$$\frac{m(\theta_2 \mid \theta_1)m(\theta_3 \mid \theta_2)\ldots m(\theta_n \mid \theta_{n-1})m(\theta_1 \mid \theta_n)}{m(\theta_n \mid \theta_1)m(\theta_{n-1} \mid \theta_n)\ldots m(\theta_2 \mid \theta_3)m(\theta_1 \mid \theta_2)} = 1. \tag{17}$$

The Markov chain with transition probabilities $p(\theta' \mid \theta)$ given by the primitive experiment p satisfies the Kolmogorov criterion (17) since p is symmetric by assumption. Finally, for any positive termination strategy β, the effective choice rule $r(\theta' \mid \theta; p, \beta, \sigma_I)$ satisfies the Kolmogorov criterion too. This is because $r(\theta' \mid \theta; p, \beta, \sigma_I) = \frac{\beta_{\theta'} p(\theta'|\theta)}{\sum_{\tilde{\theta}} \beta_{\tilde{\theta}} p(\tilde{\theta}|\theta)}$, and when the expressions for $r(\theta' \mid \theta; p, \beta, \sigma_I)$ are substituted into (17), then the terms $\beta_{\theta'}$ and the denominators cancel out, and hence

$$\frac{r(\theta_2 \mid \theta_1; p, \beta\sigma_I)r(\theta_3 \mid \theta_2; p, \beta, \sigma_I)\ldots r(\theta_1 \mid \theta_n; p, \beta, \sigma_I)}{r(\theta_n \mid \theta_1; p, \beta, \sigma_I)r(\theta_{n-1} \mid \theta_n; p, \beta, \sigma_I)\ldots r(\theta_1 \mid \theta_2; p, \beta, \sigma_I)} = \frac{p(\theta_2 \mid \theta_1)p(\theta_3 \mid \theta_2)\ldots p(\theta_1 \mid \theta_n)}{p(\theta_n \mid \theta_1)p(\theta_{n-1} \mid \theta_n)\ldots p(\theta_1 \mid \theta_2)} = 1,$$

as needed. □

Proof of Proposition 3. Lemma 9 implies for all pairs $\theta, \theta' \in \Theta$:

$$r^*(\theta \mid \theta)r^*(\theta' \mid \theta) = r^*(\theta \mid \theta')r^*(\theta' \mid \theta').$$

By Lemma 2, we can substitute $r^*(\theta' \mid \theta) = \frac{\beta^*_{\theta'} p(\theta'|\theta)}{\sum_{\tilde{\theta}} \beta^*_{\tilde{\theta}} p(\tilde{\theta}|\theta)}$, which gives

$$\frac{\beta^*_\theta \beta^*_{\theta'} p(\theta \mid \theta)p(\theta' \mid \theta)}{\left(\sum_{\tilde{\theta}} \beta^*_{\tilde{\theta}} p(\tilde{\theta} \mid \theta)\right)^2} = \frac{\beta^*_\theta \beta^*_{\theta'} p(\theta \mid \theta')p(\theta' \mid \theta')}{\left(\sum_{\tilde{\theta}} \beta^*_{\tilde{\theta}} p(\tilde{\theta} \mid \theta')\right)^2}.$$

Using symmetry of p we get

$$\frac{\sum_{\tilde{\theta}} \beta^*_{\tilde{\theta}} p(\tilde{\theta} \mid \theta')}{\sum_{\tilde{\theta}} \beta^*_{\tilde{\theta}} p(\tilde{\theta} \mid \theta)} = \frac{p^{1/2}(\theta' \mid \theta')}{p^{1/2}(\theta \mid \theta)}, \tag{18}$$

Equation (18), used below to prove the proposition, has an interesting interpretation on its own. It states that the decision rate $f_\theta = \sum_{\tilde{\theta}} \beta^*_{\tilde{\theta}} p(\tilde{\theta} \mid \theta)$ in each state θ is proportional to $p^{1/2}(\theta \mid \theta)$ and thus high in those states that are reliably identified by the primitive experiment p.

To compare $r^*(\theta_1 \mid \theta_2)$ and $r^*(\theta_2 \mid \theta_1)$, we write

$$\frac{r^*(\theta_1 \mid \theta_2)}{r^*(\theta_2 \mid \theta_1)} = \frac{\frac{\beta^*_{\theta_1} p(\theta_1|\theta_2)}{\sum_{\tilde{\theta}} \beta^*_{\tilde{\theta}} p(\tilde{\theta}|\theta_2)}}{\frac{\beta^*_{\theta_2} p(\theta_2|\theta_1)}{\sum_{\tilde{\theta}} \beta^*_{\tilde{\theta}} p(\tilde{\theta}|\theta_1)}} = \frac{\frac{\beta^*_{\theta_1} p(\theta_1|\theta_2)}{p^{1/2}(\theta_2|\theta_2)}}{\frac{\beta^*_{\theta_2} p(\theta_2|\theta_1)}{p^{1/2}(\theta_1|\theta_1)}} = \frac{\beta^*_{\theta_1} p^{1/2}(\theta_1 \mid \theta_1)}{\beta^*_{\theta_2} p^{1/2}(\theta_2 \mid \theta_2)},$$

where we have used (18) in the second step, and symmetry of p in the last step. Define $\hat{\beta}_\theta = \beta^*_\theta p^{1/2}(\theta \mid \theta)$. We need to prove that if θ_1 is more distinct than θ_2, then $\hat{\beta}_{\theta_1} > \hat{\beta}_{\theta_2}$.

By (18), $\left(\hat{\beta}_\theta\right)_\theta$ satisfy the system of linear equations:

$$\sum_{\theta'} D_{\theta'\theta} \hat{\beta}_{\theta'} = 1 \text{ for all } \theta,$$

where $D_{\theta'\theta} = \frac{p(\theta'|\theta)}{p^{1/2}(\theta'|\theta')p^{1/2}(\theta|\theta)}$. We claim that if θ_1 is more distinct than θ_2, then $D_{\theta_3\theta_1} < D_{\theta_3\theta_2}$ for all $\theta_3 \neq \theta_1, \theta_2$. This follows from $p(\theta_3 \mid \theta_1) < p(\theta_3 \mid \theta_2)$ and from the symmetry of p:

$$p(\theta_1 \mid \theta_1) = 1 - p(\theta_2 \mid \theta_1) - \sum_{\theta_3 \neq \theta_1, \theta_2} p(\theta_3 \mid \theta_1) > 1 - p(\theta_1 \mid \theta_2) - \sum_{\theta_3 \neq \theta_1, \theta_2} p(\theta_3 \mid \theta_2) = p(\theta_2 \mid \theta_2),$$

and therefore,

$$D_{\theta_3\theta_1} = \frac{p(\theta_1 \mid \theta_3)}{p^{1/2}(\theta_1 \mid \theta_1)p^{1/2}(\theta_3 \mid \theta_3)} < \frac{p(\theta_2 \mid \theta_3)}{p^{1/2}(\theta_2 \mid \theta_2)p^{1/2}(\theta_3 \mid \theta_3)} = D_{\theta_3\theta_2}.$$

Thus,

$$D_{\theta_1\theta_1}\hat{\beta}_{\theta_1} + D_{\theta_2\theta_1}\hat{\beta}_{\theta_2} = 1 - \sum_{\theta_3 \neq \theta_1, \theta_2} D_{\theta_3\theta_1}\hat{\beta}_{\theta_3} > 1 - \sum_{\theta_3 \neq \theta_1, \theta_2} D_{\theta_3\theta_2}\hat{\beta}_{\theta_3} = D_{\theta_2\theta_2}\hat{\beta}_{\theta_2} + D_{\theta_1\theta_2}\hat{\beta}_{\theta_1}.$$

Using that $D_{\theta\theta} = 1$ and $D_{\theta\theta'} = D_{\theta'\theta}$, we have

$$\hat{\beta}_{\theta_1} + D_{\theta_2\theta_1}\hat{\beta}_{\theta_2} > \hat{\beta}_{\theta_2} + D_{\theta_2\theta_1}\hat{\beta}_{\theta_1}.$$

The assumption of sufficient precision of p and symmetry of p imply that $D_{\theta_2\theta_1} < 1$, and thus $\hat{\beta}_{\theta_1} > \hat{\beta}_{\theta_2}$, as needed. □

A.4 Proofs for Section 6

Proof of Lemma 7. All rules feasible in $\mathcal{P}_{iia}$ are feasible in $\mathcal{R}(\mathcal{P}_{iia})$: $\mathcal{R}(\mathcal{P}_{iia}) \supset \mathcal{P}_{iia}$. This is immediate since when $\beta_a = 1$ for all $a \in A$, then $r(p, \beta, \sigma_I) = p$ for all $p \in \mathcal{P}_{iia}$.

It remains to show $\mathcal{R}(\mathcal{P}_{iia}) \subset \mathcal{P}_{iia}$. Consider $p(\gamma, \sigma) \in \mathcal{P}_{iia}$ constructed in the setting of example 2 by the use of the generalized termination strategy $\gamma(m, x)$, and the action strategy $\sigma(m, x)$. Recall that $r(p(\gamma, \sigma), \beta, \hat{\sigma})$ is the choice rule constructed by repetitions of the rule $p(\gamma, \sigma)$ according to the termination strategy $\beta = (\beta_a)_{a \in A}$ and by applying the action strategy $\hat{\sigma} : A \longrightarrow A$

upon the termination. We need to show that there exists γ' and σ' such that $r(p(\gamma,\sigma),\beta,\hat{\sigma}) = p(\gamma',\sigma')$. This is indeed so when the termination probability $\gamma'(\mathfrak{t} \mid m,x) = \gamma(\mathfrak{t} \mid m,x)\beta_{\sigma(m,x)}$, the transition probability to the original memory state m_0 is $\gamma'(m_0 \mid m,x) = \gamma(m_0 \mid m,x) + \gamma(\mathfrak{t} \mid m,x)\left(1-\beta_{\sigma(m,x)}\right)$, which is the sum of the probabilities that the original process γ transits to m_0 and that the decision process $r(p(\gamma,\sigma),\beta,\hat{\sigma})$ restarts after termination of $p(\gamma,\sigma)$. Additionally, for all $\tilde{m} \neq m_0$, $\gamma'(\tilde{m} \mid m,x) = \gamma(\tilde{m} \mid m,x)$. The above choice of γ' implies that the process $p(\gamma',\sigma')$ replicates the Markov process over the memory states under $r(p(\gamma,\sigma),\beta,\hat{\sigma})$. Finally, to replicate the choices upon terminations, we set the action strategy $\sigma'(m,x) = \hat{\sigma}(\sigma(m,x))$ for all (m,x). □

Proof of Lemma 8. Again, trivially, $\mathcal{R}(\mathcal{P}_{pf}) \supset \mathcal{P}_{pf}$, since $r(p,(1,\ldots,1),\sigma_I) = p$ for all $p \in \mathcal{P}_{pf}$.

Additionally, $\mathcal{R}(\mathcal{P}_{pf}) \subset \mathcal{P}_{pf}$. This is indeed so because for any $\beta = (\beta_a)_{a\in A}$ and any $\hat{\sigma} : A \longrightarrow A$, $r(p(\gamma,\sigma),\beta,\hat{\sigma}) = p(\gamma',\sigma')$ where the termination probability $\gamma'(\mathfrak{t} \mid h,\theta) = \gamma(\mathfrak{t} \mid h,\theta)\beta_{\sigma(h)}$, the transition probability to the empty signal history $\emptyset$ is set to $\gamma'(\emptyset \mid h,\theta) = \gamma(\emptyset \mid h,\theta) + \gamma(\mathfrak{t} \mid h,\theta)\left(1-\beta_{\sigma(h)}\right)$, and for all $\tilde{h} \neq \emptyset$, $\gamma'(\tilde{h} \mid h,\theta) = \gamma(\tilde{h} \mid h,\theta)$. Finally, the action strategy is set to $\sigma'(h) = \hat{\sigma}(\sigma(h))$ for all histories h. □

Proof of Proposition 4. We extend the definition of the effective experiment to the setting with discounting. Let

$$s_\delta(x \mid \theta;p,\beta) = \sum_t \sum_{\mathbf{x}^t : x_t = x} \delta^t \rho\left(\mathbf{x}^t \mid \theta;p,\beta\right),$$

where $\rho\left(\mathbf{x}^t \mid \theta;p,\beta\right)$ is the probability of the signal history $\mathbf{x}^t$ defined in (1). Thus, $s_\delta(x \mid \theta;p,\beta)$ is the discounted probability that the agent's last observed signal is x. It satisfies the recursion:

$$s_\delta(x \mid \theta;p,\beta) = \beta_x p(x \mid \theta) + \delta \sum_{x'\in X} \left(1-\beta_{x'} p\left(x' \mid \theta\right)\right) s_\delta(x \mid \theta;p,\beta), \tag{19}$$

where the first summand is the probability that the decision process terminates with x in the first round and the second summand is the discounted probability that the process terminates with x later. Solving (19) for s_δ gives

$$s_\delta(x \mid \theta;p,\beta) = \frac{\beta_x p(x \mid \theta)}{1-\delta+\delta\sum_{x'\in X}\beta_{x'} p(x' \mid \theta)}. \tag{20}$$

The discounted repeated-cognition problem (12) is thus equivalent to

$$\max_{p\in\mathcal{P},\beta\in B,\sigma\in S} \sum_{\theta\in\Theta,x\in X} \pi_\theta \frac{\beta_x p(x \mid \theta)}{1-\delta+\delta\sum_{x'\in X}\beta_{x'} p(x' \mid \theta)} u(\sigma(x),\theta). \tag{21}$$

Consider x with an interior termination probability $\beta_x^* \in (0,1)$ and let $a = \sigma^*(x)$. The first-order

condition of the problem (21) with respect to β_x is:

$$\begin{aligned}\sum_{\theta\in\Theta}\pi_\theta\frac{s_\delta(x\mid\theta;p^*,\beta^*)}{\beta^*_x}u(a,\theta)-\delta\sum_{\theta\in\Theta,x'\in X}\pi_\theta s_\delta(x'\mid\theta;p^*,\beta^*)\frac{s_\delta(x\mid\theta;p^*,\beta^*)}{\beta^*_x}u(\sigma^*(x'),\theta) &= \\ \sum_{\theta\in\Theta}\pi_\theta\frac{s_\delta(x\mid\theta;p^*,\beta^*)}{\beta^*_x}u(a,\theta)-\delta\sum_{\theta\in\Theta,a'\in A}\pi_\theta r^*_\delta(a'\mid\theta;p^*,\beta^*,\sigma^*)\frac{s_\delta(x\mid\theta;p^*,\beta^*)}{\beta^*_x}u(a',\theta) &= 0,\end{aligned}$$

where we have summed over all x' such that $\sigma^*(x')=a'$ in the second line. Multiplication by β^*_x and summation over all x such that $\sigma^*(x)=a$ gives (13). □

Proof of Proposition 5. Any (β,σ) that leads to a selection of only one action with certainty is dominated by the decision process that terminates after the first round and chooses an action equal to the observed signal. Thus, both β^*_0 and β^*_1 are positive, and the action strategy is $\sigma^*(x)=x$ or $\sigma^*(x)=1-x$. Let us show that the action strategy σ^* must be the identity function σ_I. The argument is identical to that in the proof of Lemma 5 for the setting without discounting, which we repeat here for convenience.

Assume for contradiction that $\sigma^*(x)=1-x$. The payoff difference between the rule $r_\delta(p,\beta^*,\sigma^*)$ and the choice rule that always selects $a=1$ must be positive, since the latter is dominated:

$$\begin{gathered}\pi_0u_0r_\delta(0\mid0;p,\beta^*,\sigma^*)+\pi_1u_1r_\delta(1\mid1;p,\beta^*,\sigma^*)-\pi_1u_1= \\ \pi_0u_0r_\delta(1\mid0;p,\beta^*,\sigma_I)+\pi_1u_1r_\delta(0\mid1;p,\beta^*,\sigma_I)-\pi_1u_1\geq \\ \pi_0u_0r_\delta(1\mid0;p,\beta^*,\sigma_I)-\pi_1u_1r_\delta(1\mid1;p,\beta^*,\sigma_I)>0,\end{gathered}$$

where the first inequality follows from the fact that any discounted choice rule satisfies $\sum_a r_\delta(a\mid\theta;p,\beta,\sigma)\leq 1$. Similarly, the payoff difference between the rule $r_\delta(p,\beta^*,\sigma^*)$ and the rule that always selects $a=0$ must be positive:

$$\begin{aligned}\pi_0u_0r_\delta(0\mid0;p,\beta^*,\sigma^*)+\pi_1u_1r_\delta(1\mid1;p,\beta^*,\sigma^*)-\pi_0u_0 &= \\ \pi_0u_0r_\delta(1\mid0;p,\beta^*,\sigma_I)+\pi_1u_1r_\delta(0\mid1;p,\beta^*,\sigma_I)-\pi_0u_0 &\geq \\ \pi_1u_1r_\delta(0\mid1;p,\beta^*,\sigma_I)-\pi_0u_0r_\delta(0\mid0;p,\beta^*,\sigma_I)>0.\end{aligned}$$

The last two inequalities imply:

$$\frac{r_\delta(0\mid1;p,\beta^*,\sigma_I)}{r_\delta(0\mid0;p,\beta^*,\sigma_I)}>\frac{\pi_0u_0}{\pi_1u_1}>\frac{r_\delta(1\mid1;p,\beta^*,\sigma_I)}{r_\delta(1\mid0;p,\beta^*,\sigma_I)}.$$

This establishes contradiction because as shown in the proof of Proposition 4, $r_\delta(x\mid\theta;p,\beta^*,\sigma_I)=s_\delta(x\mid\theta;p,\beta^*)=\frac{\beta^*_xp(x|\theta)}{1-\delta+\delta\sum_{x'}\beta^*_{x'}p(x'|\theta)}$, and thus

$$\frac{r_\delta(1\mid1;p,\beta^*,\sigma_I)r_\delta(0\mid0;p,\beta^*,\sigma_I)}{r_\delta(0\mid1;p,\beta^*,\sigma_I)r_\delta(1\mid0;p,\beta^*,\sigma_I)}=\frac{p(1\mid1)p(0\mid0)}{p(0\mid1)p(1\mid0)}>1.$$

Further, it must hold that $\beta_0^* = 1$ or $\beta_1^* = 1$. Otherwise, if both $\beta_0^* < 1$ and $\beta_1^* < 1$, then the agent can increase both β_x^* by a same factor. This preserves the conditional action distribution in each state θ and increases the decision rates in both states, and thus it is a profitable deviation.

Additionally, it must be that $\beta_1^* = 1$: Using the expressions for $s_\delta(\theta \mid \theta; p, \beta) = r_\delta(\theta \mid \theta; p, \beta, \sigma_I)$, the payoff for σ_I and $(\beta_0, \beta_1) = (\beta, 1)$ is

$$\pi_0 u_0 \frac{\beta\alpha}{1-\delta+\delta(\beta\alpha+1-\alpha)} + \pi_1 u_1 \frac{\alpha}{1-\delta+\delta(\alpha+\beta(1-\alpha))}, \tag{22}$$

and payoff for σ_I and $(\beta_0, \beta_1) = (1, \beta)$ is

$$\pi_0 u_0 \frac{\alpha}{1-\delta+\delta(\alpha+\beta(1-\alpha))} + \pi_1 u_1 \frac{\beta\alpha}{1-\delta+\delta(\beta\alpha+1-\alpha)}, \tag{23}$$

The assumptions that $\pi_1 u_1 > \pi_0 u_0$ and that $\alpha > 1/2$ imply that, for any $\beta \in (0, 1)$, (22) exceeds (23), as needed.

It therefore remains to find $\beta_0^* \in (0, 1]$. If the optimal value is interior, then it satisfies (13) with $a = 0$:

$$\pi_0 u_0 r_\delta(0 \mid 0, p, \beta^*, \sigma_I) = \delta \left(\pi_0 u_0 r_\delta^2(0 \mid 0; p, \beta^*, \sigma_I) + \pi_1 u_1 r_\delta(1 \mid 1; p, \beta^*, \sigma_I) r_\delta(0 \mid 1; p, \beta^*, \sigma_I) \right).$$

After the substitution of $r_\delta(x \mid \theta; p, \beta, \sigma_I) = \frac{\beta_x p(x|\theta)}{1-\delta+\delta\sum_{x'} \beta_{x'} p(x'|\theta)}$, this condition simplifies into a quadratic equation for β_0^*. When $\delta < \frac{1}{\alpha+(1-\alpha)R}$, then this condition does not have an interior solution and the derivative of the value (22) with respect to β_0 at $\beta_0 = 1$ is positive. Thus, in this case, the unique β_0^* satisfying the first-order condition is $\beta_0^* = 1$.

When $\delta > \frac{1}{\alpha+(1-\alpha)R}$, then the condition has an interior solution and the derivative of the value (22) with respect to β_0 at $\beta_0 = 1$ is negative. Thus, for this range of parameters, the unique β_0^* satisfying the first-order condition is the interior value that solves the quadratic equation, solution of which decreases in δ. □

References

Arrow, K. J., D. Blackwell, and M. A. Girshick (1949). Bayes and minimax solutions of sequential decision problems. *Econometrica 17*(3), 213–244.

Bordalo, P., N. Gennaioli, and A. Shleifer (2012). Salience theory of choice under risk. *The Quarterly Journal of Economics 127*(3), 1243–1285.

Calvert, R. L. (1985). The value of biased information: A rational choice model of political advice. *The Journal of Politics 47*(2), 530–555.

Caplin, A. and M. Dean (2015). Revealed preference, rational inattention, and costly information acquisition. *The American Economic Review 105*(7), 2183–2203.

Caplin, A., M. Dean, and J. Leahy (2017). Rationally inattentive behavior: Characterizing and

generalizing shannon entropy. Technical report, National Bureau of Economic Research.

Che, Y.-K. and K. Mierendorff (2016). Optimal sequential decision with limited attention. *unpublished, Columbia University*.

Compte, O. and A. Postlewaite (2012). Belief formation. PIER Working Paper 12-027, University of Pennsylvania.

Dow, J. (1991). Search decisions with limited memory. *The Review of Economic Studies 58*(1), 1–14.

Fryer, R. G., P. Harms, and M. O. Jackson (2016). Updating beliefs when evidence is open to interpretation: Implications for bias and polarization.

Fudenberg, D., P. Strack, and T. Strzalecki (2015). Stochastic choice and optimal sequential sampling. *unpublished, Harvard University*.

Gabaix, X. and D. Laibson (2017). Myopia and discounting. Technical report, National Bureau of Economic Research.

Gentzkow, M. and J. M. Shapiro (2006). Media bias and reputation. *Journal of Political Economy 114*(2), 280–316.

Glynn, P. W. and D. L. Iglehart (1989). Importance sampling for stochastic simulations. *Management Science 35*(11), 1367–1392.

Hébert, B. and M. Woodford (2016). Rational inattention with sequential information sampling. *unpublished, Stanford University*.

Hellmann, M. and T. M. Cover (1970). Learning with finite memory. *Annals of Mathematical Statistics 41*, 765–782.

Itti, L. (2007). Visual salience. *Scholarpedia 2*(9), 3327.

Jehiel, P. (2005). Analogy-based expectation equilibrium. *Journal of Economic Theory 123*(2), 81–104.

Jehiel, P. (2017). Investment strategy and selection bias: An equilibrium perspective on overoptimism. *unpublished, Paris School of Economics*.

Johnson, D. D., D. T. Blumstein, J. H. Fowler, and M. G. Haselton (2013). The evolution of error: Error management, cognitive constraints, and adaptive decision-making biases. *Trends in Ecology & Evolution 28*(8), 474–481.

Kahneman, D. (2011). *Thinking, fast and slow*. Macmillan.

Kahneman, D. and A. Tversky (1979). Prospect theory: An analysis of decision under risk. *Econometrica 47*(2), 263–291.

Khaw, M. W., Z. Li, and M. Woodford (2017). Risk aversion as a perceptual bias. Technical report, National Bureau of Economic Research.

Leung, T. K. (2017). Limited cognitive ability and selective information processing. *unpublished, Toulouse School of Economics*.

Lieder, F., M. Hsu, and T. L. Griffiths (2014). The high availability of extreme events serves resource-rational decision-making. In *Proceedings of the Cognitive Science Society*, Volume 36.

Luce, R. D. (1986). *Response times: Their role in inferring elementary mental organization*. Number 8. Oxford University Press on Demand.

Matějka, F. and A. McKay (2015). Rational inattention to discrete choices: A new foundation for the multinomial logit model. *The American Economic Review 105*(1), 272–298.

Meyer, M. A. (1991). Learning from coarse information: Biased contests and career profiles. *The Review of Economic Studies 58*(1), 15–41.

Morris, S. and P. Strack (2017). The wald problem and the equivalence of sequential sampling and static information costs.

Netzer, N. (2009). Evolution of time preferences and attitudes toward risk. *The American Economic Review 99*(3), 937–955.

Nickerson, R. S. (1998). Confirmation bias: A ubiquitous phenomenon in many guises. *Review of General Psychology 2*(2), 175.

Oliveira, H., T. Denti, M. Mihm, and K. Ozbek (2017). Rationally inattentive preferences and hidden information costs. *Theoretical Economics 12*(2), 621–654.

Oswald, M. E. and S. Grosjean (2004). Confirmation bias. *Cognitive illusions: A handbook on fallacies and biases in thinking, judgement and memory 79*.

Piccione, M. and A. Rubinstein (1997). On the interpretation of decision problems with imperfect recall. *Games and Economic Behavior 20*(1), 3–24.

Rabin, M. and J. L. Schrag (1999). First impressions matter: A model of confirmatory bias. *The Quarterly Journal of Economics 114*(1), 37–82.

Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review 85*(2), 59.

Ratcliff, R. and G. McKoon (2008). The diffusion decision model: theory and data for two-choice decision tasks. *Neural Computation 20*(4), 873–922.

Rayo, L. and G. S. Becker (2007). Evolutionary efficiency and happiness. *Journal of Political Economy 115*(2), 302–337.

Redish, A. D. (2016). Vicarious trial and error. *Nature Reviews. Neuroscience 17*(3), 147.

Rich, E. L. and J. D. Wallis (2016). Decoding subjective decisions from orbitofrontal cortex. *Nature Neuroscience 19*(7), 973.

Robson, A. J. (2001). The biological basis of economic behavior. *Journal of Economic Literature 39*(1), 11–33.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics 50*(3), 665–690.

Steiner, J., C. Stewart, and F. Matějka (2017). Rational inattention dynamics: Inertia and delay in decision-making. *Econometrica 85*(2), 521–553.

Suen, W. (2004). The self-perpetuation of biased beliefs. *The Economic Journal 114*(495), 377–396.

Swensson, R. G. (1972). The elusive tradeoff: Speed vs accuracy in visual discrimination tasks. *Perception & Psychophysics 12*(1), 16–32.

Wald, A. (1945). Sequential tests of statistical hypotheses. *The Annals of Mathematical Statistics 16*(2), 117–186.

Wilson, A. (2014). Bounded memory and biases in information processing. *Econometrica 82*(6), 2257–2294.

Zhong, W. (2017). Optimal dynamic information acquisition. *unpublished, Columbia University*.

Abstrakt

Agent, který čelí rozhodovacímu problému, získává informace relevantní pro zisk do okamžiku zaplnění úložní kapacity, kdy se musí rozhodnout, zda ukončit proces rozhodování a zvolit akci nebo odstranit některé informace. Agent kontroluje korelaci mezi užitkem a konečnou akcí tím, že podmiňuje pravděpodobnost ukončení získanými informacemi. Předkládáme podmínku optimality pro výslednou stochastickou volbu. Podmínka poukazuje na výhody selektivní paměti aplikované na získané signály. Omezeně optimální rozhodovací pravidlo vykazuje (i) konfirmační zkreslení, (ii) komplementaritu rychlosti a přesnosti, (iii) nadhodnocování výjimečných událostí a (iv) efekt významnost (salience).

Working Paper Series
ISSN 1211-3298
Registration No. (Ministry of Culture): E 19443

Individual researchers, as well as the on-line and printed versions of the CERGE-EI Working Papers (including their dissemination) were supported from institutional support RVO 67985998 from Economics Institute of the CAS, v. v. i.

Specific research support and/or other grants the researchers/publications benefited from are acknowledged at the beginning of the Paper.

Published by
Charles University, Center for Economic Research and Graduate Education (CERGE)
and
Economics Institute of the CAS, v. v. i. (EI)
CERGE-EI, Politických vězňů 7, 111 21 Prague 1, tel.: +420 224 005 153, Czech Republic.
Printed by CERGE-EI, Prague
Subscription: CERGE-EI homepage: http://www.cerge-ei.cz

Phone: + 420 224 005 153
Email: office@cerge-ei.cz
Web: http://www.cerge-ei.cz

Editor: Byeongju Jeong

The paper is available online at http://www.cerge-ei.cz/publications/working_papers/.

ISBN 978-80-7343-428-1 (Univerzita Karlova, Centrum pro ekonomický výzkum
a doktorské studium)
ISBN 978-80-7344-471-6 (Národohospodářský ústav AV ČR, v. v. i.)