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Abstract

We study a two-period dynamic principal agent model in which two agents with different unobservable abilities compete in a contest for a single prize. A risk-neutral principal can affect the outcome of the contest by dividing a given budget between agents in each period and her net payoff depends on the relative share of the budget given to the winner of the contest. We analyze two settings that differ by the presence/absence of moral hazard. The results we derive are consistent with stylized facts regarding the dynamics of US campaign contributions.

Keywords: Dynamic Games, Contests, Experimentation, Lobbies, Campaign Contributions
JEL Classification: D72 D78 C72 C73

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1 Introduction

An important feature that characterizes the process of campaign giving in politics is that it is an inherently dynamic process. Interest groups and donors choose to whom and when to contribute during the course of a political campaign. Furthermore, it is by no means an exception that donors choose to contribute to multiple candidates and sometimes even opposing candidates at different stages of the electoral cycle. Anecdotal evidence abounds: In the 2012 U.S. Presidential campaign, Deloitte LLP was among the top ten contributors to both Barack Obama (43% of Deloitte total contributions) and Mitt Romney (57%). Similarly, in the 2008 Presidential campaign, Citygroup Inc was among the top ten contributors to both Barack Obama (70%) and John McCain (30%). Furthermore, while a number of donors typically start contributing as early as twenty months before the elections, contributions typically peak in the three months just preceding the elections.[1]

Since the seminal contribution of Tullock (1980) on rent-seeking, the impact of money in political elections has rightly become a central theme of the political economy literature. The effect of campaign contributions on political elections has been studied under various assumptions on both the source and the use of campaign funds in election, from informative advertising to vote buying.[2] However, since existing models are essentially static in nature (the decision whether to give resources and to whom is taken once and for all), they completely abstract from the dynamic allocations of campaign funds. Our work fills this gap in the political economy literature by introducing a dynamic model of campaign contribution.

In a dynamic environment the decision of which candidate to support is naturally intertwined with the inter-temporal allocation of campaign funds between early campaign and late campaign. Such interaction raises new questions: What determines the decision of the special interest group on whether to allocate more resources early on or late in the campaign? In what circumstance does the interest group choose to contribute only to one candidate instead of giving to both? Are these two problems related?

We introduce a two-period dynamic principal-agent model to answer the questions above. The model is the simplest we could write to study the inter-temporal decision of how to allocate a fixed budget by a lobby seeking to maximize its influence on a policy. A risk-neutral principal (a lobby) chooses the amount of resources to assign to two agents with different abilities (two politicians) in each of the two periods. Agents

[1] Numbers and facts are taken from www.opensecrets.org.

[2] We review the relevant literature in the next section.

compete in a contest (an election).

We assume that the principal contributions have an important role in affecting the contest and that one of the agents has a superior technology in converting contributions into higher chances of succeeding. While it is only the outcome of the second period that determines the winner, the outcome of the first period contest may be informative of the agents abilities (and hence influence the second period allocation of the budget).[3] Finally, the principal payoff is increasing in the total contributions to the winning agent.

The environment we analyze presents a natural trade-off between experimentation and selection. In fact, the assumption that the principal payoff is increasing in the total contributions given to the winner is a force that pushes towards concentrating contributions on one agent only in the last period. On the other hand, the fact that abilities are unknown, and that first period outcomes may be informative about them, pushes towards contributing in the first period for experimentation/learning purposes.

We analyze the trade-off between experimentation and selection in two closely related models and show that they yield rather different predictions. In the first model agents make no choice and hence can be seen as different technologies the principal can experiment with. In the second model agents need to decide whether to exert or not a costly and observable effort in order to transform contributions into higher chances of succeeding. Since effort is non contractible the second model introduces a moral hazard problem.

In the model without moral hazard we find that the optimal strategy is to allocate part of the total budget to one agent only in the first period, and to allocate all remaining budget in the second period to the agent who is more likely to be the high ability one based on the posterior derived from the first period outcome. The amount provided in the first period is the result of a maximization problem that at the equilibrium optimally resolves the trade off between selection and experimentation. Notice that the principal might allocate to different agents across periods, but it is never optimal to allocate to both agents in the same period (i.e., *hedging* within a period is never optimal).

In the model with moral hazard the principal needs to incentivize agents to exert effort and therefore transform contributions into outcomes. Interestingly, the incentives to exert effort are affected by the fact that the principal can use observed effort in order to screen agents. In particular, the low ability agent has a strong incentive to mimic the high ability agent anticipating that he might be perfectly identified by the principal. In stark contrast to the model without moral hazard, this force leads to the non-existence

[3]For example, we can interpret it as the result of an electoral poll.

of equilibria where the principal allocates any amount of budget to only one of the two agents.

In the model with moral hazard only strategies that either allocate to both agents in the first period or that allocate all the budget in the second period to one agent only may be optimal. Note that, within the set of strategies that allocate to both agents in the first period, both perfect screening strategies and strategies that provide information based on the first period outcome may be incentive compatible. This is somewhat surprising because a screening strategy perfectly separates the agents and it is therefore superior in terms of experimentation to any other strategy. We characterize which of the above mentioned strategies is optimal as a function of the effort cost.

Our model yields rich predictions that are consistent with a number of facts pertaining to the dynamics of US campaign contributions. For example, as we argued earlier, data shows that it is not uncommon that lobbies contribute to both candidates in an electoral campaign. To the best of our knowledge, we are not aware of other theoretical explanations that would support this observation and which is not based on risk attitude.

We keep our model deliberately stylized so that it can be applied directly, or with small modifications, to a number of different applications. For example, the model may capture equally well a labor market relation where the employer is uncertain about the ability of competiting workers and uses an intermediate outcome produced by the agents to choose which agent to retain. The employer choice is how to assign training resources among employees and across periods. Notice that depending on the specific applications, the contribution can be though of as money or any other scarce resource held by the principal's such as time. Finally, the framework can be used to study conflict problems where an outside party chooses strategically which side in a conflict to support and when.

2 Related Literature

Our work is related to several strands of literature. First, we review the literature on our leading application of dynamic campaign contributions. Then, we move on describing how our model relates to existing papers on experimentation.

One of the earliest examples of modeling the influence of money in election is Tullock (1980). His rent-seeking model studies the spending competition between interest groups or lobbies over the allocation of a political favor. Snyder (1989) studies campaign expenditure allocation between parties competing in different districts. Grossman and Helpman (1996) model the influence of campaign contribution in a common agency framework a-la Bernheim and Whinston (1986) where campaign advertising buys the support of noise-voters. The literature has also explored the idea that rational voters observe platforms with errors. Austen-Smith (1987) shows that informative campaign advertising can be beneficial to risk-averse voters. Coate (2004), Prat (2002a) and Prat (2002b) study the direct and indirect informative role of campaign contributions, respectively. Contrary to our work, this literature focuses on purely static models.[4]

A second related strand of literature focuses on the efficiency consequences of vote buying. In Dekel et al. (2008) parties alternate in a sequential bidding process in which they offer either campaign promises or upfront vote buying. While the purposes of this literature is very different from our research goal, we share the idea that elections can be modeled as markets for votes and assume that different politicians can be viewed as different technologies to transform campaign money in electoral support.

Our work is also related to the literature on social learning and momentum building in sequential elections. Knight and Schiff (2010) empirically document a substantial momentum effect focusing on 2004 Democratic primaries. Klumpp and Polborn (2006) show theoretically that victory in an early primary increases the likelihood of victory in the general election and creates an important asymmetry in campaign spending.[5] Contrary to our theoretical framework, they do not model lobbying effort.

In terms of theoretical underpinning, our work is also related to the existing literature on dynamic contracts, experimentation and contests. Here we review only a few papers that like ours lie at the intersection of those, and point out how they differ from our. Our main results that the principal might want to contribute to both agents in the early period is reminiscent of a result in Halac et al. (2017). They find that under some conditions a hidden equal-sharing contest dominates a public winner-takes-all contest. Their model, however, differs in several dimension from ours: agents do not know their probabilities of success; their principal, unlike in our model, has control over the disclosure policy of the contest; the principal cannot boost the chances of success of an agent. Thus, in their model there is no trade off between learning via experimentation and the risk of boosting the electoral prospects of the "inferior" candidate, which is a key element in our setting. In fact, our reason for contributing to both agents comes from a very different argument as we mentioned earlier in the introduction and will come back in detail later. Another interesting paper dealing with the incentives for

[4] See also Brams and Davis (1973, 1974), Stromberg (2008), Iaryczower and Mattozzi (2013), Denter (2013), and Meirowitz (2008).

[5] See also Denter and Sisak (2014).

experimentation is Hörner and Samuelson (2013). However, it focus on one agent only, and our argument for the optimality of hedging contributions comes from the fact that there are at least two agents.

3 The Model

Players

We have one principal P (she) and two agents A_i (he), $i \in \{1, 2\}$ that compete over two periods, $t = 1, 2$. The principal chooses how much of her total budget $B > 1$ to allocate to each agent across the two periods.[6] We call B_i^t the amount allocated to agent i in period t, and we define $B_i \equiv B_i^1 + B_i^2$ the total contributions to agent i by the principal, and by $B^t \equiv \{B_1^t, B_2^t\}$ the vector of contributions at time t. The amount contributed to agents in any period is common knowledge.

Agents compete in a two-periods contest. We consider two different models that differ depending on whether moral hazard is present or not. If moral hazard is not present the agents are effectively technologies and our model resembles more closely a model of experimentation. However, differently from the experimentation literature, our focus is on the strategic effects due to inter-temporal allocation of a scarce resource. In the model with moral hazard, instead, agents choose whether to exert effort or shirk in each period, where $e_i^t = \{0, 1\}$ denotes agent i's effort in period t. Each period effort costs an amount C to the agent, and we assume that effort is ex-post observable but it is not contractible.

In each period, each agent may produce either a failure (denoted by 0) or a success (denoted by 1), that is the binary outcome $O_i^t \in \{0, 1\}$ denotes the output of agent i in period t. We define $O^t \equiv \{O_1^t, O_2^t\}$ the vector of realized outputs at time t.[7] The winner of the contest is the agent who succeeds in the second and last period. In case of a tie, each agent has an equal probability of winning. Every player is risk neutral.

Conditional on exerting effort, the probability of success for each agent i in period t depends only on the principal's contribution B_i^t and on his type. In particular, when agent i receives an amount of resources B_i^t in period t, his probability of obtaining a success in period t is $\theta_i f(B_i^t)$, where we assume that $\theta_i f(B) < 1\ \forall i$, and that $f(\cdot)$ is strictly increasing and strictly concave, and $f(0) = 0$ and $f'(0) \to \infty$.[8] One of

[6] If the total budget is not large enough, the principal will never invest in experimentation. As we will shown in the proof of Lemma 2, this is the case when $B < 1$.

[7] In our political economy example, one can interpret these outcomes as polls conducted before the election and on the election day, which generate binary signals.

[8] We assume that both agents depend on the principal's budget, i.e., the probability of generating a

the two agents is always more productive than the other. In particular, we assume that the most productive agent has a type $\theta > 0$, and we set the productivity of the least productive agent to be equal to zero.[9] The principal does not observe agents' productivities, and he has a symmetric prior probability that each agent is the most productive.

The main difference between the model without moral hazard and the model with moral hazard is that in the latter case a success can be obtained only if agents voluntary exert effort, while in the former $e_i^t = 1$ by assumption.

Finally, we define by $\lambda_i(O^1, B^1) \in [0, 1]$ the principal's posterior belief at the end of period $t = 1$ that agent i is the more productive one.[10] Note that $\lambda_i(O^1, B^1)$ has an important role in our analysis. It measures the precision of the posterior, which is affected by the first period budget allocation B^1 and the first period outcome O^1. Since O^1 can only take values $\{0, 0\}$, $\{0, 1\}$ (or $\{1, 0\}$), the first period contribution choice B^1 maps into a binary outcome. Furthermore, the only case in which the principal does not have deterministic beliefs about the agents is after $\{0, 0\}$. These facts make our analysis more tractable.

Despite its simplicity the model remains rich enough to capture the fact that the choice of first period contributions impacts "smoothly" the amount of learning that is achieved at the end of period one. Indeed, the level of B^1 affects the relative likelihood of reaching the outcomes $\{0, 0\}$ and $\{0, 1\}$ (or $\{1, 0\}$) as well as the beliefs conditional on observing $\{0, 0\}$. Our equilibrium concept is PBE.

Timing

The timing of the game is as follows (in brackets the part that is present only in the moral hazard model):

1. Agents observe their private type

2. Principal chooses first-period contributions

3. (Agents choose simultaneously whether to exert effort or not)

success for an agent that did not receive a contribution is zero. Alternatively, we could have assumed that $f(0) > 0$. This specifications would not change the results if $f(0)$ is small enough. If $f(0)$ is large, instead, a waiting strategy in which the principal free rides on agents in the first period may be optimal.

[9] This case provides most of the intuition for our main mechanism, which generalizes to the case where the least productive agent has also a positive but small productivity.

[10] Given our previous interpretation of early and late outcomes, see footnote 7, λ_i could be also seen as the principals' posterior after having observed a poll.

4. Successes (and efforts) are observed

5. Principal chooses second-period contributions

6. (Agents choose simultaneously whether to exert effort or not)

7. Payoffs are collected

Payoffs

The utility of the principal depends on her contributions to the agent winning the contest relative to her contribution to the losing one. Specifically, the expected payoff for the principal is

$$\Pi(B_i, O_i^2) \equiv \begin{cases} g(B_i)W & \text{if } O_i^2 = 1 \text{ and } B_i \geq B/2 \\ 0 & \text{otherwise,} \end{cases}$$

where $g(B_i) \rightarrow [0,1]$, $g(B_i) = 0$ for $B_i < B/2$, $g(B/2) = \varepsilon > 0$, and $g(B) = 1$. Furthermore, we assume that $g(\cdot)$ is concave and strictly increasing in its argument for $B_i \geq B/2$. The idea is that the principal can receive a prize (e.g. a favorable policy, profits from a patent) from the winning agent, and this is worth $W > 0$ to her. However, the probability of receiving the prize depends on the principal relative contributions to agents, and it is described by the $g(\cdot)$ function. In the case in which the principal has contributed an equal amount to each agent, we assume that the probability of receiving a prize is vanishingly small, i.e., we consider the limiting case in which ε approaches zero.[11] Finally, we assume that early and late contributions are weighted equally and that in case of a tie the principal does not collect W.[12]

The utility of an agent is irrelevant for the principal perspective in the model without moral hazard. For the model with moral hazard, we assume that it depends on whether the agent wins the contest or not and on the effort exerted. Specifically, the payoff for a winning agent i is

$$V - (I_i^1 + I_i^2)C,$$

where V is the payoff of winning the contest and $I_i^t = 1$ if agent i exerted effort in period t and zero otherwise. On the other hand, the losing agent $(-i)$ gets $-(I_{-i}^1 + I_{-i}^2)C$.

[11]We bound away from zero the value of $g(\cdot)$ at $B/2$ for technical reasons. Without it, we would have a trivial case of non-existence of equilibrium when the optimal lopsided strategy is given by a corner solution, see Lemma 2.

[12]This assumption is done for simplicity and can be relaxed.

4 A Model of Experimentation and Selection

In our model, the principal wants to allocate resources to the most likely winner of the contest. How does this affect her incentives to contribute in each period of the game?

Contributing early not only affects the probability of collecting a higher payoff through the effect on $g(\cdot)$, but it also generates information on who the most productive agent is. In particular, first period contributions have two competing effects for the principal. The positive effect is that the more she contributes early, the better informed she will be when deciding how to allocate the remaining of the budget in the second period. The negative effect is that the more the principal contributes in the first period, the less resources are available to generate a success in the second period. Furthermore, if the principal decides to favor one of the two agents in the first period by allocating him relatively more budget, she needs to consider that the gains from switching to favoring the other agent in the second period might diminish. All these elements introduce a trade-off between experimentation and selection.

We study this trade-off in two relevant cases. We first consider the case in which moral hazard is absent. In this case, as we mentioned earlier, we assume that agents are *de facto* different technologies, and the principal has to choose on which technology allocating her budget across periods. Then, we consider the case in which the principal must provide incentives for the agents to exert effort.

In terms of strategies, we refer to a *lopsided* (contribution) strategy if the principal contributes a strictly positive amount only to one agent in a given period. We refer to a *hedging* (contribution) strategy if the principal contributes a strictly positive amount to both agents in a given period. Within hedging strategies we further distinguish between *symmetric hedging* when the principal contributes a strictly positive and equal amount to both agents, and *asymmetric hedging* otherwise. Finally, we define a *betting* strategy as a degenerate lopsided strategy such that the principal allocates the entire budget to one agent only in the second period.

4.1 Second Period Budget Allocation

Most of the action of our model happens in the first period, the one in which experimentation takes place. Indeed, the optimal strategy in the second period does not entail any trade off. The next result, which applies independently of whether moral hazard is present or not, characterizes the optimal strategy in the second period.[13]

[13]The proof can be found in the Appendix.

Result 1 *The optimal strategy in the second period is to allocate all unspent budget to one agent only. In particular, the principal allocates to* A_i *if* $\lambda_i(O^1, B^1) \geq g(B - B_i^1)/(g(B - B_i^1) + g(B - B_{-i}^1))$*, and to* A_{-i} *otherwise.*

The result follows from the observation that the second period choice is equivalent to selecting which agent ends up with a total budget larger than $B/2$, and that once the selection is made, the principal payoff is increasing in the amount allocated to that agent. From now on we will say that the principal strategy *selects* agent A_i, when it is such that more than half of the total budget is allocated to agent A_i. We are now ready to consider the first period and study the properties of the model in the two cases, depending on whether moral hazard is present or not.

4.2 Experimentation without Moral Hazard

In this case, if the principal decides to give an amount B_i^t to agent A_i in period t, he has a positive probability of returning a success - equal to θB_i^t - if and only if he chooses the productive agent. The payoff of the principal is influenced by the way she allocates the budget *across* periods and *between* agents within periods.

The equilibrium can be characterized using backward induction. We already solved for the second-period optimal decision conditional on any first-period allocation of budget and on the intermediate outcome. Next, in Lemma 1, we show some properties of the optimal first-period decision, which will be characterized in Lemma 2. In Proposition 1, we characterize the equilibrium.

Lemma 1 *It is never optimal for the principal to contribute to both agents in the first period.*

Proof. See the Appendix. ■

To see the intuition for this result consider that, for any hedging strategy, we can construct a lopsided contribution strategy that allocates the same amount as the hedging strategy does to one of the agents in the first period and nothing to the other agent. This strategy always dominates the hedging strategy. In fact, whenever the hedging strategy selects one agent, the constructed lopsided strategy selects the same agent and allocates the same total budget in some continuation play and strictly more in others. Intuitively, the key aspect is that the amount that the lopsided strategy "saves" from the first period is allocated under a superior information in the second period.

Now we turn to the determination of the optimal lopsided contribution in the first period by first establishing the existence of an upper bound on how much to contribute in the first period.

Lemma 2 *There exists a unique* $\hat{B}_i^1 \in (0, B/2]$ *increasing in* θ *and* B *such that a necessary condition for a lopsided strategy* B_i^1 *to be optimal is* $B_i^1 < \hat{B}_i^1$.

Recall that if the principal gives a lopsided contribution in the first period, it must be that following a $(0,0)$ (i.e., two failures), she switches contributing to the other agent. This is because if the principal does not switch after $(0,0)$ she would never, which would be dominated by flipping a coin and giving the entire budget in the second period.

Consider a lopsided contribution $(0, B_2^1)$, the expected payoff of the principal after a $(0,0)$ outcome if she keeps contributing to the same agent A_2 is equal to

$$\lambda_2(0, 0; B_1^1 = 0, B_2^1 > 0)\theta f(B - B_2^1)W$$

where

$$\lambda_2 = \frac{1 - \theta f(B_2^1)}{2 - \theta f(B_2^1)} < \frac{1}{2}$$

is the principal's posterior belief that A_2 is the most productive agent conditional on the observed outcome and given the contribution schedule $(0, B_2^1)$. On the other hand, the expected payoff of the principal after a $(0,0)$ outcome if she switches and contributes to the other agent A_1 is equal to

$$(1 - \lambda_2)\theta f(B - B_2^1)g(B - B_2^1)W$$

It is immediate to see that when B_2^1 gets close to $B/2$ (and $\varepsilon \to 0$) switching is never optimal. On the other hand, when B_2^1 gets close to zero, both payoffs become identical. The condition for the principal to switch is

$$\lambda_2 = \frac{1 - \theta f(B_2^1)}{2 - \theta f(B_2^1)} < \frac{g(B - B_2^1)}{1 + g(B - B_2^1)}$$

which can be rewritten as

$$1 - \theta f(B_2^1) < g(B - B_2^1)$$

The larger $\hat{B}_i^1$ is, the larger the set of values for which it is optimal to switch contributing to the agent who initially got no resources. The comparative statics with respect to θ and B then have a clear interpretation. If B is larger, there is more budget to be allocated following two failures, which gives more value to the option of switching contributions. Similarly, a larger θ makes switching contributions more valuable because it implies both a better posterior after observing two failures as well as a higher chance of generating a success conditional on having switched in favor of the high type.

Combining our results we can finally characterize the equilibrium of our model in the absence of Moral Hazard in the next proposition.

Proposition 1 *In the unique equilibrium of the game the principal makes an optimal lopsided contribution $\tilde{B}_i^1 \in (0, \hat{B}_i^1)$ to agent i in the first period and then keeps contributing to the same agent if and only if she observes a success.*

The optimal strategy is the solution to the following problem

$$\max_{B_i^1 \in [0, \hat{B}_i^1]} \frac{\theta W}{2} \left(\theta f(B_i^1) f\left(B - B_i^1\right) + f\left(B - B_i^1\right) g\left(B - B_i^1\right) \right)$$

The solution is the level of B_i^1 that resolves optimally the trade-off between experimentation and selection.

Proposition 1 follows as a straightforward corollary of the results shown in the previous lemmas. In particular, when $\tilde{B}_1^i < \hat{B}_1^i$, since $\tilde{B}_1^i$ is the solution to the principal's maximization problem, then it must be the case that a lopsided contribution is better than contributing all budget to one of the two agents.

In the next section we introduce moral hazard and show that in this case it might be optimal for the principal to contribute to both agents in the first period.

4.3 Experimentation with Moral Hazard

In this section we characterize the equilibrium of the model with moral hazard, and show that it delivers rather different insights. In our analysis we introduce the assumption that $C < V\theta f(B)/2$. This is a sufficient condition to guarantee that for the low type, who can only tie the contest by assumption, exerting effort in the first period is not a dominated strategy.[14] This condition will also imply that for the high type exerting effort is not dominated.

Our model is perhaps the simplest one could write where the relevant dynamic trade-offs are presents. Still, its dynamic features imply that characterizing equilibria involves checking a number of potential deviations in different continuation games. Below we prove a series of lemmas that restrict the type of deviations we need to check and are therefore preliminary facts we need to prove before stating our main results.

We will begin with Lemma 3 showing that a lopsided contribution strategy cannot be sustained as an equilibrium. This result restricts the set of potential equilibrium

[14] If we relax the simplifying assumption that the low type can never obtain a success we can weaken this requirement.

strategies to either a betting strategy or a hedging strategy. Next, Lemma 4 restricts the type of hedging contributions that may be supported as an equilibrium strategy. In particular, it shows that either we may have a symmetric hedging strategy that induces both agent to exert effort, or a hedging strategy that induce only the high type to exert effort, and hence implements perfect screening. Lemma 6 further shows that for any asymmetric hedging strategy we can find a symmetric hedging that yields a larger expected payoff for the principal. Then, Lemma 5 and Lemma 7 analyze the screening and the non screening scenario. Finally, Proposition 2 specifies under what conditions each of the candidate equilibrium strategies that have been identified are optimal.

An important difference with the model without moral hazard is that the principal may use first period contributions to screen the agents, and that screening is a more effective way to learn agents types than bayesian updating based on the first period outcome. The possibility of screening, however, creates a strong incentive for the low type to mimic the high type in order not to be identified. This incentive is one of the forces that drives the non existence of equilibria based on lopsided contributions. We state this result in the next lemma.

Lemma 3 *A lopsided contribution strategy cannot be supported as an equilibrium.*

The intuition for this result is simple. A first-period lopsided contribution can be supported in equilibrium only if at least one type of agent exerts effort in the first period. If it is exactly one type, it must be the high type. But then the low type finds it profitable to mimic the high type. If instead both types are supposed to exert effort then, conditional on observing a failure, the principal's posterior tilts in favor of the agent who did not receive the contribution. In this case, independently of out-of-equilibrium beliefs, the low type always wants to deviate.

The next lemma restricts the type of hedging strategies that can be part of an equilibrium.

Lemma 4 *Consider any hedging contribution strategy* (B_1^1, B_2^1) *such that* $B_1^1 > 0$ *and* $B_2^1 > 0$*. Then, the strategy* (B_1^1, B_2^1) *can be supported in equilibrium only if the high type exerts effort in the first period. Furthermore, the low type exerts effort in equilibrium only if* $B_1^1 = B_2^1$*.*

Lemma 4 has two immediate consequences. First, a screening strategy in which the low type agent may exert effort cannot be supported in equilibrium. Any such strategy would induce the worse equilibrium outcome for the low type. In fact, since the opponent receives the entire remaining budget, the low type is better off deviating and saving the effort cost. Second, an asymmetric hedging strategies such that both agents exert effort in equilibrium violates incentive compatibility. The reason is that the principal, upon observing two failures, would allocate all remaining budget to the agent who received the smaller contribution. This implies that if the low type agent received the larger contribution he is sure that he will not get any budget in the second period. Hence, he is better off deviating and saving the effort cost.

Within the set of hedging strategies we are thus left with two possible candidates: a hedging strategy such that only the high type exerts effort, and a symmetric hedging strategy such that each agent exerts effort regardless of his type. The first type of strategy, which can be either asymmetric or symmetric, induces perfect screening of the agents types.

We first consider hedging strategies that ensure screening. For this we need that the cost of effort is such that the low type does not have an incentive to exert effort, while the high type does. It turns out that at the optimal symmetric hedging screening strategy, it is only the upper bound on C that matters.

Lemma 5 *The optimal symmetric hedging screening strategy is* $B_1^1 = B_2^1 = \frac{B^S}{2}$*, where* B^S *the unique solution to*

$$\frac{4C}{\theta V} = \left(1 - f\left(\frac{B^S}{2}\right)\theta\right) f(B - B^S)$$

Furthermore, $B^S > 0$ *if and only if* $C < f(B)\theta V/4$*.*

The result follows directly from the fact that the proposed strategy minimizes - among the set of feasible strategies - the amount of first period budget needed to ensure that only the high type exerts effort in the first period. Since at the optimal strategy the low type is indifferent between exerting effort or not, a fortiori the high type is willing to exert effort even for a very low cost.

Corollary 1 *The payoff for the principal in an optimal symmetric hedging screening strategy is strictly increasing in* C*.*

The corollary presents the perhaps surprising comparative statics result that when moral hazard becomes more severe (C increases) the principal becomes better off. To see why it holds notice that the condition that determines the optimal first period contribution

$$\frac{4C}{\theta V} = \left(1 - f\left(\frac{B^S}{2}\right)\theta\right) f(B - B^S)$$

implies that as C increases the principal can screen agents with a lower level of first period contributions B^S. Hence, more budget is available to be allocated in the second period to the high ability agent. An intuitive way to see where the result comes from is to observe that if C is low, the principal has to discourage the low type from exerting effort by increasing B^S. In fact, by doing so the principal increases the chance that the high ability type succeeds in the first period.

Next, fixing the total level of first period contributions at B^S, let us consider the case of an *asymmetric* hedging screening strategy (B_1^1, B_2^1) such that $B_1^1 + B_2^1 = B^S$, and let $B_1^1 > B_2^1 > 0$ without loss of generality. For incentive compatibility, we need to make sure that the low type does not want to deviate independently from the contribution received. In this case it is important to specify out of equilibrium beliefs and we assume that if the principal observes both agents exerting effort and two failures, her posterior is equal to the prior. This seems a natural assumption that we will maintain throughout.[15] Given this assumption, we have that screening can occur via an asymmetric hedging strategy if:

$$\left(1 - f(B - B^S)\theta\right) \frac{V}{2} \geq \left(1 - f(B_2^1)\theta\right) \frac{1}{2}\frac{V}{2} +$$

$$\left(1 - f(B_2^1)\theta\right) \frac{1}{2} \left(1 - f(B - B^S)\theta\right) \frac{V}{2} + f(B_2^1)\theta(1 - f(B - B^S)\theta)\frac{V}{2} - C$$

Since the right hand side of the inequality is decreasing in B_2^1, the inequality holds also for $B_1^1 > B_2^1$. Furthermore, $B_1^1 + B_2^1 = B^S$ implies that $B^S > B_2^1$, hence it must be the case that if C is such that the low type does not want to deviate in the case of an asymmetric hedging strategy, a fortiori he has no incentive to deviate to exert effort in the case of a symmetric hedging contribution strategy. In other words, whenever an asymmetric hedging contribution strategy leading to screening can be supported in equilibrium so does a symmetric hedging contribution strategy. Given our assumption on out of equilibrium beliefs, the following result holds.

Lemma 6 *For any asymmetric hedging strategy with screening there exists a symmetric hedging strategy with the same total level of first period contributions, which yields a strictly larger expected payoff for the principal.*

We now move to the case of a symmetric hedging strategy such that each agent exerts effort regardless of his type. Since both agents exert effort, screening is not feasible any more, and the characterization of the optimal strategy is the result of a

[15] Alternative specifications are clearly possible but would not alter substantially our results.

maximization problem, which resembles the case without moral hazard. For that we also need to define what the principal believes when agents choose to shirk in the first period. We will assume that out-of-equilibrium beliefs are such that the deviator is considered a low type with probability one. These out of equilibrium beliefs satisfy the intuitive criterion. Let B^{NS} the total value of first period contributions in a symmetric non-screening (NS) hedging equilibrium. In the next lemma we characterize the optimal strategy within the class of non-screening symmetric hedging.

Lemma 7 *Consider a non-screening symmetric hedging strategy and assume out of equilibrium beliefs surviving the intuitive criterion. Incentive compatibility requires* $B^{NS} \leq \bar{B}^{NS}$*, and* $\bar{B}^{NS} \geq B^{NS} > 0$ *if and only if* $f(B)\theta V/4 > C$*. Furthemore, for* C *small enough,* $B^{NS} < \bar{B}^{NS}$*.*

Incentive compatibility, see Lemma 4, requires that the first period contribution strategy is such that the low type (and hence the high type) prefers to choose high effort rather than low effort. This requirement delivers the upper bound $\bar{B}^{NS}$ on the total value of first period contributions B^{NS}. By inspecting the condition that determines $\bar{B}^{NS}$,

$$\frac{V}{4}\left(1 - f\left(\frac{B^{NS}}{2}\right)\theta\right) f\left(B - B^{NS}\right)\theta = C$$

it is immediate to see that the upper bound decreases (it is more tight) when either C is larger or V is smaller. Both changes correspond to an aggravated moral hazard. In this case, to preserves the incentives for the low type it cannot be that the high type could produce a success with a too large probability and hence $\bar{B}^{NS}$ has to decrease. Finally, $\bar{B}^{NS}$ increases with θ. To see why, note that the low type needs to compare the expected profits of the proposed equilibrium strategy with those of shirking, which are different from zero only if the high type does not generate a success in the first period. The likelihood of the latter event decreases in θ, which corresponds to a less severe moral hazard problem and hence as θ increases more resources can be allocated to first period learning.

We are now ready to characterize the equilibrium of the model with moral hazard in the next proposition.

Proposition 2 *Assume out of equilibrium beliefs that survive the intuitive criterion. There exists a unique* $C' < f(B)\theta V/4$ *such that equilibrium first period contributions are:*

i) symmetric non screening hedging, if $C < C'$*;*

ii) symmetric screening hedging, if $C \in (C', f(B)\theta V/4)$*;*

iii) betting, if $C \geq f(B)\theta V/4$*.*

Each of the candidates for an optimal strategy that we had previously determined is indeed the equilibrium strategy for some space of the parameter C. Notice that a betting strategy is optimal only when it is the only strategy that can be supported in equilibrium. It is not surprising that this is the case when moral hazard is the most severe. More interestingly, both a screening symmetric hedging strategy and a non screening symmetric hedging strategy are optimal for some value of C. In fact, given that a screening strategy perfectly identifies types, it is not obvious that other strategies can be optimal given that the screening strategy is incentive compatible whenever the non screening is. By looking at Lemma 5, however, we can see that the optimal amount of first period contributions needed to make screening incentive compatible approaches the total budget B as C approaches zero. This implies that while screening is feasible even for arbitrarily small C, in that case the payoff of the principal is minimized, since there is no budget left to generate a success in the second period.

The other key observation is that while the payoff of the principal in a non-screening strategy remains constant or decreases as C increases (it decreases if the incentive constraint becomes binding), the payoff of the principal under screening increases in C as argued in Corollary 1. As a result, for small values of C non-screening is optimal and for larger values of C screening is optimal provided that it is incentive compatible.

5 Conclusions and Discussion

We have presented a dynamic model of campaign contribution. Our simple framework allows to investigate how the inter-temporal allocation of resources interacts with the decision of which agent to support. The model can also be applied to a labor market relation where the employer is uncertain about the ability of the workers and uses an intermediate outcome produced by the agents to choose which agent to retain. In fact, our work may describe any dynamic principal-two agents contest in which the principal can influence the outcome of the contest by allocating a scarce resource over two periods.

The key modeling assumptions are that the principal does not know ex-ante the identity of the more productive agent, and that the payoff of the principal is increasing in the amount allocated to winner. We have shown that this creates an interesting trade off between experimentation and selection and we have analyzed two variants of the model: one without moral hazard, and one with moral hazard. The model, despite being stylized, has rich implications that well describe commonly observed facts in campaign contributions. In particular, it shows under which conditions the strategies used by lobbies are optimal, including the fact that sometimes lobbies contribute to opposing politicians. This latter prediction is hard to reconcile with available models, unless one is willing to introduce risk aversion.

There are a number of extensions of the basic model worth considering. In the basic model we assumed that agents know their opponent type. Assuming that each agent does not know the type of the opponent would not affect the analysis substantially. In that case each agent would be playing against the average type and this is equivalent to compressing the value of θ. More importantly, the principal's incentives and therefore her strategy would not be qualitatively affected. A different situation would be to consider a model in which agents do not know their own type. In this case, exerting effort in the first period would disclose valuable information to the agents themselves. This case, however, leads to a rather different type of model closer in spirit to the classic models of experimentation.

Another simplifying assumption is that the agents cannot *roll over* contributions across periods. In other words, each agent exerting effort in a given period uses all available B_i^t. On the other hand, upon shirking, all B_i^t for period t are foregone. Relaxing this assumption would imply that agents could potentially undo the principal's strategy. In particular, a high type agent receiving contributions in the first period might have an incentive to use less resources in the first period and use the remaining budget in the last period. This will affect the incentive compatibility constraints that the principal faces in the first period. As a result, some higher level of first period contributions might no longer be optimal and the principal would give only an amount that she anticipates will be fully spent. We also assume that the principal commits to exhaust the overall budget over the two periods. By relaxing this assumption, the moral hazard would be exacerbated and the principal might need higher first period contributions for incentive compatibility to hold.

Finally, a more fundamental departure is to add an additional principal and allow for competition between principals. In this case one would expect that each agent would get contributions. However, it is unclear under which conditions each principal would either give lopsided contributions to one agent or hedge and give to both. We leave these extensions for future research.

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Appendix

Proof. of Result 1 Assume first that either $B_1^1 > B/2$ or $B_2^1 > B/2$. Then it is optimal for the principal to allocate all remaining budget to the agent that had more than $B/2$ in period 1. In fact, giving to the other agent yields zero for sure, while the proposed strategy yields a strictly positive expected payoff regardless of the history. Now assume that $B_2^1 \leq B/2$ and $B_1^1 \leq B/2$. In this case, the principal choice determines who of the two agents should end up with a total budget greater than $B/2$. How much the other agent receives is irrelevant for the principal payoff. But then given that the principal payoff is increasing in the share allocated to the winner, we can conclude that the agent who received a total budget above $B/2$ should receive all budget not allocated in period 1. To determine the identity of who gets the contribution, the condition in the lemma simply compares the expected payoff of giving to A_i and A_{-i} given the information available at the end of period 1. ■

Proof. of Lemma 1

To prove the lemma, we argue that any hedging contribution in the first period, that is any (B_1^1, B_2^1) with $B_1^1 > 0$, $B_2^1 > 0$ is dominated by a lopsided contribution. To do so assume (w.l.o.g) that $B_2^1 \geq B_1^1$. Consider first the case in which $B_2^1 > B/2 > B_1^1 > 0$. This type of contribution is such that, regardless of the history that unfolds, the principal does not get any payoff from having allocated $B_1^1 > 0$ to A_1. Hence, she is strictly better off by setting $B_1^1 = 0$ as this increases her payoff conditional on A_2 winning and does not affect it conditional on A_1 winning.

Consider next any hedging contribution in the first period such that $B/2 > B_2^1 \geq B_1^1 > 0$. We want to show that for any such strategy we can find a lopsided strategy that dominates it. In particular the lopsided we propose depends on whether the considered hedging strategy is such that after observing two failures the principal contributes to A_1 (case a) or to A_2 (case b) in the second period. These are the only relevant cases since after observing one success the principal must optimally allocate all remaining budget to the agent who succeeded

Case a) We compare the hedging (B_1^1, B_2^1) with a lopsided contribution $(0, B_2^1)$ in which the principal contributes to A_1 in the second period after observing two failures. Since the principal cannot distinguish ex-ante between the two agents, we compare the payoffs under the two strategies depending on whether A_2 is the high type. First, the situation in which A_2 is the high type. Suppose that with $(0, B_2^1)$ a success is observed. Then the second period decision is to give all remaining budget to A_2 under both type of contribution strategies. However, $B_1^1 > 0$ has two negative effects: it reduces the probability of getting a prize because $g(B) > g(B - B_1^1)$, and it reduces the probability of generating a success in the second period. Suppose instead that with $(0, B_2^1)$ a failure is observed. Then, both strategies contribute all remaining budget to A_1, which yield a zero payoff for both (the contest ends with a sure draw).

Second, the situation in which A_1 is the high type. In this case, the second period decision is always to give all remaining budget to A_1, so that the total budget share allocated to A_1 is the same under the two strategies, which means that the resulting value for $g(\cdot)$ is the same. Furthermore, the lopsided contribution strategy has a higher chance of generating a success in the last period because the second period contribution is now $B - B_2^1 > B - B_2^1 - B_1^1$. Hence the lopsided strategy yields a higher payoff.

Thus, we can conclude that any hedging strategy of the type considered in case a) is dominated by the proposed lopsided strategy.

Case b) We compare the hedging (B_1^1, B_2^1) with a lopsided contribution $(B_1^1, 0)$ in which the principal contributes to A_2 in the second period after observing two failures. The proof is identical to case a), it only requires to invert the roles of A_1 and A_2. ■

Proof. of Lemma 2

We now show that there exists a bound on B_2^1 smaller then $B/2$ such that the first period contribution of an optimal lopsided strategy will be always smaller then this bound. Recall that the condition for the principal to switch is

$$\lambda_2 = \frac{1 - \theta f(B_2^1)}{2 - \theta f(B_2^1)} < \frac{g(B - B_2^1)}{1 + g(B - B_2^1)}$$

which can be rewritten as

$$1 - \theta f(B_2^1) < g(B - B_2^1)$$

Consider then the function

$$Q(B_2^1) \equiv 1 - \theta f(B_2^1) - g(B - B_2^1)$$

Notice that $Q(0) = 0$ and $Q(B/2) > 0$. Furthermore,

$$Q'(B_2^1) = -\theta f'(B_2^1) + g'(B - B_2^1),$$

$$Q'(0) = -\theta f'(0) + g'(B) < 0$$

and

$$Q''(B_2^1) = -\theta f''(B_2^1) - g''(B - B_2^1) > 0.$$

Hence, there exists a unique positive $\hat{B}_2^1 < B/2$ such that $Q(B_2^1) < 0$ if and only if $B_2^1 < \hat{B}_2^1$. It follows that the optimal lopsided contribution is to give some $B_2^1 \leq \hat{B}_2^1$ (since any $B_2^1 > \hat{B}_2^1$ implies that the principal will keep contributing to the same agent no matter what the outcome is at the end of the first period).
The comparative static results follow from straightforward application of the implicit function theorem to $Q(\hat{B}_2^1) = 0$. Indeed, existence of $\hat{B}_2^1$ implies that $Q'(\hat{B}_2^1) > 0$. Hence

$$\frac{dQ(\hat{B}_2^1)}{dB} = -\frac{-g'(B - \hat{B}_2^1)}{Q'(\hat{B}_2^1)} > 0$$

and

$$\frac{dQ(\hat{B}_2^1)}{d\theta} = -\frac{-f(\hat{B}_2^1)}{Q'(\hat{B}_2^1)} > 0$$

■

Proof. of Proposition 1

Conditional on giving a lopsided contribution in the first period, the optimal strategy in this case is the solution to the following problem

$$\max_{B_2^1 \in [0, \hat{B}_2^1]} \frac{\theta W}{2} \left(\theta f(B_2^1) f\left(B - B_2^1\right) + f\left(B - B_2^1\right) g\left(B - B_2^1\right)\right)$$

Taking the first order conditions we get

$$\theta(f'(B_2^1)f(B - B_2^1) - f(B_2^1)f'(B - B_2^1)) - f'(B - B_2^1)g(B - B_2^1) - f(B - B_2^1)g'(B - B_2^1),$$

and notice that the FOC evaluated at $\hat{B}_2^1$ is equal to

$$f(B - \hat{B}_2^1)\left(\theta f'(\hat{B}_2^1) - g'(B - \hat{B}_2^1)\right) - f'(B - \hat{B}_2^1) < 0$$

since by the uniqueness of $\hat{B}_2^1$ it must be that

$$Q'(\hat{B}_2^1) = -\theta f'(\hat{B}_2^1) + g'(B - \hat{B}_2^1) > 0.$$

Furthermore, by evaluating the FOC at $B_2^1 = 0$ yields

$$\theta f'(0)f(B) - f'(B) - f(B)g'(B) > 0$$

Evaluating the objective function at the boundaries we get

$$\frac{\theta W}{2} f(B)$$

at $B_2^1 = 0$ and

$$\frac{\theta W}{2} f\left(B - \hat{B}_2^1\right)\left(\theta f(\hat{B}_2^1) + g\left(B - \hat{B}_2^1\right)\right) = \frac{\theta W}{2} f\left(B - \hat{B}_2^1\right) < \frac{\theta W}{2} f(B)$$

at $B_2^1 = \hat{B}_2^1$, where the equality follows from the definition of $\hat{B}_2^1$.
Hence, the optimal lopsided strategy is $B_2^1 > 0$. Finally, taking the second order condition yields

$$\theta\left(f''(B_2^1)f(B - B_2^1) - 2f'(B_2^1)f'(B - B_2^1) + f(B_2^1)f''(B - B_2^1)\right) +$$

$$+f''(B - B_2^1)g(B - B_2^1) + 2f'(B - B_2^1)g'(B - B_2^1) + f(B - B_2^1)g''(B - B_2^1),$$

which is not necessarily negative since $2f'(B - B_2^1)g'(B - B_2^1)$ is positive. Hence, uniqueness of an optimal lopsided strategy requires imposing further conditions on the concavity of $f(\cdot)$ and $g(\cdot)$. ■

Proof. of Lemma 3

Assume that a lopsided strategy can be supported in equilibrium. We show that there always exist a profitable deviation either by the principal or the agent. First, assume that the candidate lopsided strategy is such that the agent who receives a positive contribution in the first period does not exert effort regardless of his type. Then a deviation of the principal to not contributing at all in the first period is profitable as it achieves the same first period outcome and same posterior beliefs, but saves resources for the second period and thus raises the expected payoff of the principal. Second, assume that the agent who gets the contribution exerts effort only if he is the high type. In this case the principal, by observing the effort choice would infer the agent type. For the principal's actions to be consistent with such deterministic beliefs, it must be that following two failures in the first period she allocates all residual budget to the agent who received the first period contribution if he exerted effort, and to the other agent otherwise. Consider then a deviation of the low type agent to exerting effort when he receives the contribution. In this case, the agent is sure to collect all remaining budget in period 2, which corresponds to the best possible outcome for him. Indeed, he is certain of achieving a draw with the high type, which is the best outcome from his perspective. This is a profitable deviation if and only if $C < V/2$, which is implied by our maintained assumption that $C < V\theta f(B)/2$. Finally, assume that the agent who receives the contribution exerts effort regardless of his type. Consider then a deviation of the low type agent to shirk. Upon deviating, the low type saves on the effort cost C and, depending on out-of-equilibrium beliefs, he may even receive $B - B_i^1$ in the second period. In this latter case, the low type will tie the contest for sure, and he will shirk in the second period. Instead, by following the candidate equilibrium, the low type receiving the contribution exerts effort and he fails for sure, in which case the remaining budget goes to the other agent. This is so because both types are expected to exert effort in equilibrium so conditional on observing a failure the posterior tilts in favor of the other agent. Hence, independently of out-of-equilibrium beliefs the low type wants to deviate. ■

Proof. of Lemma 4

We proceed in steps.

First, notice that any hedging contribution strategy such that no type of agent exerts effort is dominated by the principal not contributing at all in the first period.

Second, any hedging strategy such that the only event in which effort is exerted is when the high type receives the larger contribution cannot be an equilibrium. In fact, in this case the low type can profitably deviate and exert effort when receiving the larger contribution. By deviating he is identified as the high type and achieves the best possible outcome, i.e., he obtains all remaining budget in the second period for sure.

Third, we show that any hedging strategy such that both agents exert effort if the high type receives the smaller contribution, while only the high type agent exerts effort if he receives the smaller contribution is not an equilibrium. Indeed, the low type not exerting effort when he receives the larger contribution is a profitable deviation. To see why this is the case, note that by playing the equilibrium strategy types are perfectly revealed - which ties the principal to deterministic beliefs - and hence the low type does not receive any contribution in the second period. By deviating, the low type saves the effort cost and cannot do worse than not receiving any budget for sure, independently of out of equilibrium beliefs.

Fourth, we show that any asymmetric hedging strategy such that each agent exerts effort regardless of his type and regardless of whether he received the larger or the smaller contribution is not part of an equilibrium. To see why, take any such strategy and consider first the case in which following two failures, the principal contributes all remaining budget to the agent that received the larger contribution. In other words, the principal behaves “against her posterior”. In this case, for any out of equilibrium belief, we have that the principal benefits from deviating to a betting strategy. This follows from the fact that the value of contributing in the first period is only due to learning. Hence, acting against the posterior after two failures implies that the principal would allocate the (smaller) second period budget less efficiently than what she would have done with a betting strategy. Consider then the case in which following two failures, the principal contributes all remaining budget to the agent that received the smaller contribution. Then the low type has a profitable deviation: not exerting effort when receiving the larger contribution. In this way he saves the cost of effort and cannot do worse than not receiving any budget in the second period, which is the equilibrium path outcome.

Hence we are left we only two possibilities for a (candidate) equilibrium hedging strategy: i) a symmetric hedging strategy such that each agent exerts effort regardless of his type; ii) a hedging strategy such that only the high type exerts effort. ■

Proof. of Lemma 5

If we define B^S to be the total level of first period contributions in a symmetric hedging equilibrium, we have that screening can occur via a symmetric hedging strategy if:

$$\left(1 - f(B - B^S)\theta\right)\frac{V}{2} \geq \left(1 - f\left(\frac{B^S}{2}\right)\theta\right)\frac{1}{2}\frac{V}{2} +$$
$$\left(1 - f\left(\frac{B^S}{2}\right)\theta\right)\frac{1}{2}\left(1 - f(B - B^S)\theta\right)\frac{V}{2} + f\left(\frac{B^S}{2}\right)\theta(1 - f(B - B^S)\theta)\frac{V}{2} - C$$

The left hand side is the equilibrium payoff of the low type agent if he follows the equilibrium strategy. The right hand side is the payoff of the low type agent if he deviates to exerting effort. In this case if the principal observes two failures he has no way of distinguishing the two types and the out of equilibrium beliefs coincide with the prior. The above inequality simplifies to

$$C \geq \frac{\left(1 - f\left(\frac{B^S}{2}\right)\theta\right) f(B - B^S)\theta V}{4}$$

Furthermore, the high type exerts effort in the first period if and only if

$$f(B - B^S)\theta V + \left(1 - f(B - B^S)\theta\right)\frac{V}{2} - 2C \geq \frac{V}{2}\left(f(B - B^S)\theta + (1 - f(B - B^S)\theta)\frac{1}{2} + \frac{1}{2}\right) - C$$

which yields

$$f(B-B^S) \geq \frac{4C}{\theta V} \text{ or } C \leq \frac{\theta V}{4} f(B-B^S)$$

Hence, for $C \in \left(\frac{\left(1-f\left(\frac{B^S}{2}\right)\theta\right) f(B-B^S)\theta V}{4}, \frac{f(B-B^S)\theta V}{4} \right)$ a symmetric hedging strategy can be supported in equilibrium. Clearly, the optimal symmetric hedging minimizes the total level of contribution B^S in the first period, and hence equals the unique solution to

$$\frac{4C}{\theta V} = \left(1 - f\left(\frac{B^S}{2}\right)\theta\right) f(B-B^S),$$

which is strictly positive if and only if $C < \frac{f(B)\theta V}{4}$. Note that, at the optimal B^S, it is always the case that

$$f(B-B^S) \geq \frac{4C}{\theta V}$$

since $1 - f\left(\frac{B^S}{2}\right) < 1$. ■

Proof. of Lemma 6

Fix the level of total first period contribution B^S. Assuming passive out of equilibrium beliefs, incentive compatibility for the low type of an asymmetric hedging strategy (B_1^1, B_2^1), such that $B_1^1 + B_2^1 = B^S$ and $B_1^1 > B_2^1$ without loss of generality, requires

$$\left(1 - f(B-B^S)\theta\right)\frac{V}{2} \geq \left(1 - f\left(B_1^1\right)\theta\right)\frac{1}{2}\frac{V}{2} +$$

$$\left(1 - f\left(B_1^1\right)\theta\right)\frac{1}{2}\left(1 - f(B-B^S)\theta\right)\frac{V}{2} + f\left(B_1^1\right)\theta(1 - f(B-B^S)\theta)\frac{V}{2} - C$$

and

$$\left(1 - f(B-B^S)\theta\right)\frac{V}{2} \geq \left(1 - f\left(B_2^1\right)\theta\right)\frac{1}{2}\frac{V}{2} +$$

$$\left(1 - f\left(B_2^1\right)\theta\right)\frac{1}{2}\left(1 - f(B-B^S)\theta\right)\frac{V}{2} + f\left(B_2^1\right)\theta(1 - f(B-B^S)\theta)\frac{V}{2} - C$$

Since, at the optimal (B_1^1, B_2^1) one of the two inequalities above will be binding, this implies that the incentive compatibility constraint for the low type of a symmetric hedging strategy will be slack, i.e.,

$$\left(1 - f(B-B^S)\theta\right)\frac{V}{2} > \left(1 - f\left(\frac{B^S}{2}\right)\theta\right)\frac{1}{2}\frac{V}{2} +$$

$$\left(1 - f\left(\frac{B^S}{2}\right)\theta\right)\frac{1}{2}\left(1 - f(B-B^S)\theta\right)\frac{V}{2} + f\left(\frac{B^S}{2}\right)\theta(1 - f(B-B^S)\theta)\frac{V}{2} - C$$

Hence, to establish which of the two strategies dominates we only need to compare the principal expected payoff of a symmetric hedging strategy that allocates B^S in the first period with a asymmetric hedging that allocates (B_1^1, B_2^1) with $B_1^1 + B_2^1 = B^S$. The comparison gives:

$$\frac{1}{2}g(B - B_1^1) + \frac{1}{2}g(B - B_2^1) < g\left(B - \frac{B^S}{2}\right),$$

where the left hand side is the payoff under the asymmetric strategy and the right hand side the payoff under the symmetric strategy. The inequality stated holds because of the assumed concavity of $g(\cdot)$. ■

Proof. of Lemma 7

Given the previous lemmas we only need to determine the total value of first period contributions in a symmetric non-screening (NS) hedging equilibrium, which we define as B^{NS}. By Lemma 4 we need to ensure that the first period contribution strategy is such that the low type (and hence the high type) prefers to choose high effort rather than low effort. In doing so, we assume punitive out-of-equilibrium beliefs, that is beliefs such that if an agent shirks in the first period, he is considered a low type with probability one. Hence, we need

$$\frac{V}{2}\left(1 - f\left(\frac{B^{NS}}{2}\right)\theta\right)\left[\frac{1}{2} + \frac{1}{2}\left(1 - f\left(B - B^{NS}\right)\theta\right)\right] +$$
$$\frac{V}{2}f\left(\frac{B^{NS}}{2}\right)\theta\left(1 - f\left(B - B^{NS}\right)\theta\right) - C \geq \frac{V}{2}\left(1 - f\left(B - B^{NS}\right)\theta\right),$$

which simplifies to

$$\frac{V}{4}\left(1 - f\left(\frac{B^{NS}}{2}\right)\theta\right)f\left(B - B^{NS}\right)\theta \geq C$$

The condition is satisfied when evaluated at $B^{NS} = 0$ (it simplifies to $f(B)\theta V/4 \geq C$, which is a necessary condition for a symmetric screening hedging equilibrium to exist). Furthermore, its left hand side is strictly decreasing for all range of feasible B^{NS}, and the condition is clearly violated at $B^{NS} = B$. Hence, a unique solution to the inequality above exists for each positive C, and it yields an upper bound on B^{NS}. We call such upper bound $\bar{B}^{NS}$. Next, we compute the optimal level of B^{NS}. The principal maximizes

$$\max_{B^{NS} \leq \bar{B}^{NS}} W\left(1 - f\left(\frac{B^{NS}}{2}\right)\theta\right)\frac{1}{2}f\left(B - B^{NS}\right)\theta g\left(B - \frac{B^{NS}}{2}\right) +$$

$$Wf\left(\frac{B^{NS}}{2}\right)\theta f\left(B-B^{NS}\right)\theta g\left(B-\frac{B^{NS}}{2}\right)$$

which simplifies to

$$\max_{B^{NS}\leq \bar{B}^{NS}} \frac{\theta W}{2} f(B-B^{NS}) g\left(B-\frac{B^{NS}}{2}\right)\left(1+f\left(\frac{B^{NS}}{2}\right)\theta\right)$$

Taking first order conditions yields

$$-f'(B-B^{NS}) g\left(B-\frac{B^{NS}}{2}\right)\left(1+f\left(\frac{B^{NS}}{2}\right)\theta\right)+$$

$$-\frac{1}{2} f(B-B^{NS}) g'\left(B-\frac{B^{NS}}{2}\right)\left(1+f\left(\frac{B^{NS}}{2}\right)\theta\right)+$$

$$+\frac{1}{2} f(B-B^{NS}) g\left(B-\frac{B^{NS}}{2}\right) f'\left(\frac{B^{NS}}{2}\right)\theta$$

Evaluating the first order conditions at $B^{NS}=0$, we have that a sufficient condition for a strictly positive B^{NS} is

$$\frac{f'(0)\theta - g'(B)}{2} > \frac{f'(B)}{f(B)},$$

which always holds given that $f'(0)\to\infty$. Furthermore, when C is relatively small, $\bar{B}^{NS}$ approaches B and evaluating the first order conditions at $B^{NS}=B$ yields

$$-f'(0)\varepsilon < 0$$

which implies that when C is small, the optimal B^{NS} is strictly smaller than B. ■

Proof. of Proposition 2

First notice that for small C, a S strategy cannot be optimal. To see this notice that the optimal B^S, as proved in Lemma 5, is the unique solution to

$$\frac{4C}{\theta V} = \left(1-f\left(\frac{B^S}{2}\right)\theta\right) f(B-B^S)$$

and as C approaches zero, B^S approaches B. Hence the principal payoffs in a S strategy for small C approaches zero. First notice that a NS strategy - whenever it is incentive compatible - dominates a betting strategy since the optimal B^{NS} is strictly positive. In this case, the optimal B^{NS} does not depend on C as long as $B^{NS} < \bar{B}^{NS}$, i.e., the optimal B^{NS} is strictly interior, which also implies that the principal's payoff does not depend on C (and if B^{NS} hits the upper bound the profit can only decrease). On the other hand, the principal's payoff in a S strategy increases with C, since the principal can attain screening using less resources in the first period. Since for C close to $f(B)\theta V/4$ the principal attains the highest possible profit, namely she can achieve screening spending almost no resources in the first period, there must exist a level of $C' < f(B)\theta V/4$ such that a S strategy dominates any other possible strategy for $C \in (C', f(B)\theta V/4)$. Finally, when $C \geq f(B)\theta V/4$ the only (trivially) incentive compatible strategy is a betting strategy. ■

Abstrakt

Tento článek zkoumá model pán-agent (principal-agent). Jedná se o model, ve kterém dva ekonomičtí agenti s různými schopnostmi soutěží o jednu cenu. Pán s neutrálním přístupem k riziku může ovlivnit výsledek této soutěže tím, že rozděluje prostředky mezi oba agenty v každém ze dvou období modelu, přičemž se snaží maximalizovat prostředky přidělené tomu agentovi, který nakonec soutěž vyhraje. Analyzovaný model má dvě verze, které se liší tím, zda zahrnují morální hazard či nikoliv. Predikce modelu jsou konzistentní s řadou faktů o dynamice volebních příspěvků v USA.

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